

# Modelling the Rough Piston Skirts EHL at a Small and a Large Radial Clearance in the Initial Engine Start Up

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## ABSTRACT

*At initial engine start-up, the low speeds take time to develop considerable film thickness between piston and liner surfaces. Factors like radial clearance, oil viscosity and roughness of interacting surfaces play an important role in this regard. A two dimensional model is presented for hydrodynamic lubrication and EHL of rough piston skirt under isothermal conditions. Flow factors method is used to incorporate the deterministic isotropic roughness effects in Reynolds' equation and Greenwood-Tripp asperity contact model is used for involving the corresponding asperity contact forces and moments. The numerical model is used to get simulation results including piston eccentric displacements, hydrodynamic and EHL film thickness and rising pressures for 10 micron and 100 micron radial clearance between piston and liner. All results are plotted against 720 degree crank rotation cycle. The results show considerable change in each parameter by changing radial clearance from small value to large.*

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## 1. INTRODUCTION

The surface roughness of an engineered surface is classified as: the random surface roughness which is a product of surface finishing and the deterministic surface roughness which is induced to increase the lubrication between interacting surfaces [1]. In 1966 the effect of deterministic roughness characteristics on lubrication of interacting parallel surfaces was first investigated by Hamilton et al. [2]. The presented theory does provide a lubrication

model for rough surfaces but the classical lubrication theory does not help in finding the pressure existence on any of the interacting surfaces. Thus without the pressure development idea on rough surfaces, the application of model was limited. The effect of sliding rough surface on load carrying capacity was investigated by Tzeng et al. [3] and an increase in load carrying capacity by lubrication film was concluded by their presented model. The increase in the total load carrying capacity is more significant as it leads to a lower coefficient

of friction by avoiding the physical contact between surfaces. But the lubrication model was limited to one dimension only for hydrodynamic sliding bearing. Then in 1969, Christensen [4] applied the stochastic roughness concept in hydrodynamic bearing. The one dimensional model helps in investigating the directional behaviour of surface roughness. Results depicts that the roughness effects can be neglected, if the mean separation of the sliding surfaces is large compared to the height of roughness asperities, and conversely roughness effects becomes prominent when surfaces are close to each other. An exception and the major research is presented in 1978 and 1979 by Patir and Cheng [5,6]. The model is based on flow factors which were introduced to modify the three dimensional average Reynolds Equation. In the PC formulation, an averaged flow between nominally smooth surfaces is assumed to approximate the actual flow between rough surfaces, while parameters describing the roughness are included in the Reynolds equation through the flow factors. The model was developed for hydrodynamic lubrication i.e.  $\lambda > 3$  where  $\lambda$  is film thickness parameter [7]. But initially it provides best estimation of roughness effects on fluid film lubrication in all regimes in context to the Peklinik number and dimensionless film thickness parameter. In 1983, J.H Tripp [8] provided the modified flow factors based upon perturbation theory. The flow factors by Tripp are available for  $\lambda < 3$ , i.e. the EHL and mixed lubrication regimes can also be covered as the  $\lambda$  range is redefined by Zhu et al. [9]. Several researchers using flow factor method to provide solution of roughness effects on slider bearings. In 1987, B.J. Hamrock et al [10] provide theoretical study of surface roughness in EHL point contact. In 1989, Sadeghi and Sui [11] presented one dimensional compressible EHL lubrication model of rough surfaces. A numerical solution using Newton-Raphson method presented with high level of accuracy. Compressibility was proved to be significant parameter and low loads with roughness orientation were investigated. In 1991, Dong Zhu et al [12] used the flow factor method to provide a two dimensional solution of mixed lubrication in case of piston and liner lubrication for mixed and boundary lubrication regimes at hot running conditions. Greenwood et al (1994) [13] and Morlas-Espejel (1996) [14] investigated the effects of transverse surface

roughness in EHL. Lunde and Tonder [15], in 1997, calculated the flow factor for an isotropic rough bearing. In 2012, Zhu et al. [9] presented the dimensionless film thickness parameter range for EHL in rough surfaces, which redefined the  $\lambda$  range. Greenwood- Tripp's asperity contact model [16] is used in the presented model for modelling the rough piston motion characteristics and estimating the role of asperities contacts. But as a matter of fact all the above mentioned researchers provided models for hot running conditions. In this paper, the initial engine start-up speed of 600 rpm is focussed for rough piston lubrication using different bore to cylinder radial clearances (10 micron and 100 micron), and a comparison is provided in results.

## 2. NOMENCLATURE

$C$  = Radial clearance between piston and liner = 10microns, 100 micron

$C_f$  = Specific heat of lubricant,

$C_g$  = Distance from piston center of mass to piston pin = 0.2cm,

$C_p$  = Distance of piston-pin from axis of piston = 1 cm,

$F$  = Normal force acting on piston skirts,

$F_f$  = Friction force acting on skirts surface,

$F_{fh}$  = Friction force due to hydrodynamic lubricant film,

$F_G$  = Combustion Gas force acting on the top of piston,

$F_h$  = Normal force due to hydrodynamic pressure in film,

$F_{IC}$  = Transverse Inertia force due to piston mass,

$\tilde{F}_{IC}$  = Reciprocating Inertia force due to piston mass,

$F_{IP}$  = Transverse Inertia force due to piston pin mass,

$\tilde{F}_{IP}$  = Reciprocating Inertia force due to piston pin mass,

$F_c$  = Asperity Contact Force,

$F_{fc}$  = Friction force due to asperity contact,

$G$  = Shear modulus of elastic lubricant,

$I_{pis}$  = Piston inertia about its centre of mass,

$M$  = Moment acting on piston skirts,

$M_f$  = Friction moment acting on skirt surface,

$M_{fh}$  = Moment about piston pin due to hydrodynamic friction,

$M_h$  = Moment about piston pin due to hydrodynamic pressure,

$M_c$  = Asperity Contact Moment,

$M_{fc}$  = Moment due o friction force of asperity contact,  
 $R$  = Radius of piston,  
 $U$  = Piston Velocity,  
 $a$  = Vertical distance from skirt top to piston-pin = 0.0125m,  
 $b$  = Vertical distance from skirt top to piston center of gravity = 0.0015m,  
 $e_t$  = Piston eccentricities at skirts top surface,  
 $e_b$  = Piston eccentricities at skirts bottom surface,  
 $\ddot{e}_b$  = Acceleration of piston skirts bottom eccentricities,  
 $\ddot{e}_t$  = Acceleration of piston skirts top eccentricities,  
 $h$  = Film Thickness,  
 $l$  = Connecting rod length,  
 $m_{pis}$  = Mass of piston = 0.295 ,  
 $m_{pin}$  = Mass of piston-pin = 0.09 kg,  
 $p$  = Hydrodynamic pressure,  
 $r$  = Crank radius = 0.0418 m,  
 $\omega$  = Constant crankshaft speed (engine speed),  
 $\tau$  = Shear stress,  
 $\eta$  = Oil viscosity = 0.1891 Pa.s.,  
 $\Phi$  = Connecting rod angle,  
 $\psi$  = Crank angle,  
 $\phi_x, \phi_y$  = Pressure flow factor along x and y-axis respectively,  
 $\phi_s$  = Shear flow factor,  
 $\sigma$  = combined root mean square (rms) roughness,  
 $\sigma_1$  = rms roughness of piston skirt= 1.4 $\mu$ m,  
 $\sigma_2$  = rms roughness of cylinder liner = 1.5 $\mu$ m,

### 3. MATHEMATICAL MODEL

#### 3.1 PISTON SECONDARY MOTION:

In order to understand the motion of piston the mathematical model presented by Zhu et al [12] can be applied:

$$U = \dot{Y}r \omega \sin \psi + r \omega B \cos \psi (l^2 - B^2)^{0.5} \quad (1)$$

$$B = C_p r \sin \psi$$

$$\begin{bmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \ddot{e}_t \\ \ddot{e}_b \end{bmatrix} = \begin{bmatrix} F_h + F_c + F_s + (F_{fh} + F_{fc}) \tan \Phi \\ M_h + M_c + M_s + M_f \end{bmatrix} \quad (2)$$

$$a_{11} = m_{pin} \left( 1 - \frac{a}{L} \right) + m_{pin} \left( 1 - \frac{b}{L} \right) \quad (2a)$$

$$a_{12} = m_{pin} \left( \frac{a}{L} \right) + m_{pin} \left( \frac{b}{L} \right) \quad (2b)$$

$$a_{21} = \left( \frac{I_{pin}}{L} \right) + m_{pin} (a - b) \left( 1 - \frac{b}{L} \right) \quad (2c)$$

$$a_{22} = m_{pin} (a - b) \left( \frac{b}{L} \right) - \left( \frac{I_{pin}}{L} \right) \quad (2d)$$

$$F_s = \tan \Phi (F_G + \tilde{F}_{IP} + \tilde{F}_{IC}) \quad (3)$$

$$M_s = F_G C_p + \tilde{F}_{IC} C_g \quad (4)$$

By using Greenwood-Tripp asperity contact model [20] for EHL regime average contact forces and moments are found.

#### 3.2 LUBRICATION MODEL:

Lubrication film thickness between piston and cylinder can be given as [12]:

$$h = C + e_t(t) \cos x + \left[ e_b(t) - e_t(t) \right] \frac{y}{L} \cos x \quad (5)$$

similarly for EHL film thickness:

$$h_{ehl} = h + f(\theta, y) + v \quad (6)$$

where  $f(\theta, y)$  is manufacturing imperfections and neglected in this model. The differential surface displacement is [18]:

$$dv = \frac{1}{\pi E'} \frac{p(x, y) dy dy}{\dot{r}} \quad (7)$$

$$\dot{r} = \sqrt{(x-x_0)^2 + [(y-y_0)^2]} \quad (8)$$

$$\frac{1}{E'} = \frac{1}{2} \left[ \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \right] \quad (9)$$

At a specific point  $(x_0, y_0)$  the elastic deformation is [7]:

$$v(x_0, y_0) = \frac{1}{\pi E'} \iint_a \frac{p(x, y) dx dy}{\dot{r}} \quad (10)$$

Two dimensional Reynolds equation with P.C. flow factors is given as [5]:

$$\frac{\partial}{\partial x} \left( h^3 \phi_x \frac{\partial p}{\partial x} \right) + \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial y} \left( h^3 \phi_y \frac{\partial p}{\partial y} \right) = 6U\eta \left( \frac{\partial h_r}{\partial x} + \sigma \frac{\partial \phi_s}{\partial x} \right) \quad (11)$$

where  $\phi_x$  and  $\phi_y$  are Poiseuille flow factors and  $\phi_s$  is shear flow factor [5,8]. The boundary conditions are defined as [7]:

$$\frac{\partial p}{\partial x_{\theta=0}} = \frac{\partial p}{\partial x_{\theta=\pi}} = 0 \tag{12}$$

$$p=0 \quad \text{when } x_1 \leq x \leq x_2$$

$$p(\theta, 0) = p(\theta, L) = 0$$

In dimensionless form the 2-D Reynolds equation is given by [9][7]:

$$\frac{\partial}{\partial x^*} \left( h^{*3} \phi_x \frac{\partial p^*}{\partial x^*} \right) + \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial y^*} \left( h^{*3} \phi_y \frac{\partial p^*}{\partial y^*} \right) = \frac{\partial h_r^*}{\partial x^*} + \sigma^* \frac{\partial \phi_s}{\partial x^*} \tag{13}$$

Where by J. H Tripp [8]

$$\phi_x = 1 + [3(\gamma - 2)/(\gamma + 1)][\sigma/h]^2 \quad \phi_y = \phi_x (1/\gamma) \tag{14}$$

$$\phi_s(\gamma/\sigma, \gamma_2) = \left[ \frac{3}{(\gamma + 1)} (\sigma/h) \right] \tag{15}$$

$$\phi_s(\gamma/\sigma, \gamma_2) = \left[ \frac{3}{(\gamma + 1)} (\sigma/h) \right] \tag{16}$$

$$\phi_s = \frac{\sigma_1^2}{\sigma^2} \phi_s(\gamma/\sigma, \gamma_1) - \frac{\sigma_2^2}{\sigma^2} \phi_s(\gamma/\sigma, \gamma_2) \tag{17}$$

and  $\gamma$  is the Peklenik number [21]. In order to read the pressure profiles conveniently, the Vogelpohl parameter  $M_v = p^* h^{*1.5}$  is introduced [7]. The Reynolds equation in terms of the Vogelpohl parameter is given as:

$$\frac{\partial^2 M_v}{\partial x^{*2}} + \left( \frac{R}{L} \right)^2 \frac{\partial^2 M_v}{\partial y^{*2}} + \left( \frac{\partial \phi_x}{\partial x^*} (1/\phi) \right) \left( \frac{\partial M_v}{\partial x^*} \right) + \left( \frac{\partial \phi_y}{\partial y^*} (1/\phi) \right) \left( \frac{R}{L} \right)^2 \left( \frac{\partial M_v}{\partial y^*} \right) = FM_v + G \tag{18}$$

where

$$F = \frac{0.75 \left[ \left( \frac{\partial h^*}{\partial x^*} \right)^2 + \left( \frac{R}{L} \right)^2 \left( \frac{\partial h^*}{\partial y^*} \right)^2 \right]}{h^{*2}} + \frac{1.5 \left[ \frac{\partial^2 h^*}{\partial x^{*2}} + \left( \frac{R}{L} \right)^2 \frac{\partial^2 h^*}{\partial y^{*2}} \right]}{h^*} + \frac{1.5 \left[ \frac{\partial h^*}{\partial x^*} \left( \frac{\partial \phi}{\partial x^*} \right) + \left( \frac{R}{L} \right)^2 \frac{\partial \phi}{\partial y^*} \left( \frac{\partial h^*}{\partial y^*} \right) \right]}{h^* \times \phi}$$

$$G = \frac{\left( \frac{\partial h^*}{\partial x^*} + \sigma^* \frac{\partial \phi_s}{\partial x^*} \right)}{(h^{*1.5} \times \phi)}$$

$$M_{v,i,j} = \frac{C_1 (M_{v,i+1,j} + M_{v,i-1,j}) + \left( \frac{R}{L} \right)^2 C_2 (M_{v,i,j+1} + M_{v,i,j-1})}{2.C_1 + 2.C_2 + F_{i,j}} + \frac{C_3 \left( \frac{\partial \phi}{\partial x^*} (1/\phi^*) \right) (M_{v,i+1,j} - M_{v,i-1,j})}{2.C_1 + 2.C_2 + F_{i,j}} + \frac{C_4 \left( \frac{\partial \phi}{\partial y^*} (1/\phi^*) \right) (M_{v,i,j+1} - M_{v,i,j-1}) - G_{i,j}}{2.C_1 + 2.C_2 + F_{i,j}}$$

where

$$C_1 = \frac{1}{\delta x^{*2}} \quad C_2 = \frac{1}{\delta y^{*2}}$$

$$C_3 = \frac{1}{2\delta x^*} \quad C_4 = \frac{1}{2\delta y^*}$$

and shear stress is given as:

$$\tau = - \frac{\mu U}{h} (\Phi_f + \Phi_{fs}) + \Phi_{fp} \frac{h}{2} \frac{dp}{dx} \tag{19}$$

The factors  $\Phi_f$ ,  $\Phi_{fs}$  and  $\Phi_{fp}$  are given by Zhu et al. [12] and are used for roughness only with no waviness effects.

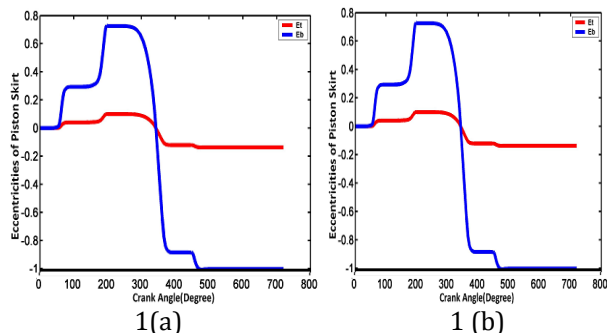
#### 4. RESULTS AND DISCUSSION

At initial engine start-up speeds the parameters affecting the lubrication phenomenon, includes lubricant viscosity, start-up speeds, radial clearance and surface roughness. The effect of viscosity on rough surfaces lubrication has been investigated in the previous research and it was concluded that low viscosity oil is more suitable for EHL formation at initial start-up speeds [19]. Now in the following simulations the effects of radial clearance in two dimensional piston skirt lubrication model is demonstrated for two different cases. Case 1, represents radial clearance of 10 micron, and case 2 represents radial to bore clearance of 100 microns for SAE 50 oil at 600 rpm engine start-up speed.

##### 4.1 RADIAL CLEARANCE OF 10 MICRON

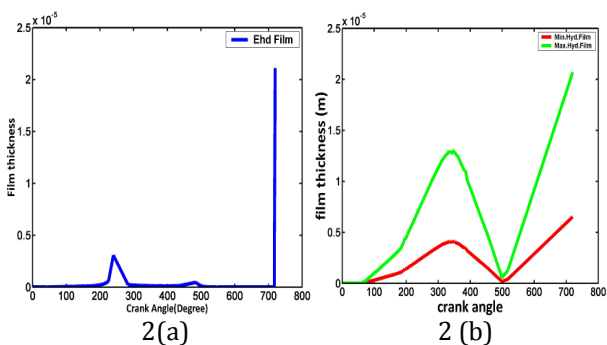
The dimensionless eccentricities for a radial clearance of 10 micron are shown in Fig. 1. The upper and lower lines related to values of 1 and -1 are showing the thrust and non thrust side respectively. The '0' value indicates the piston and liner axis to be concentric. Now in Fig. 1(a) for hydrodynamic regime, it is clearly evident that the eccentric displacements are such that

they are touching the value of -1, i.e. lower line which shows that the physical contact cannot be avoided between piston and liner surfaces.



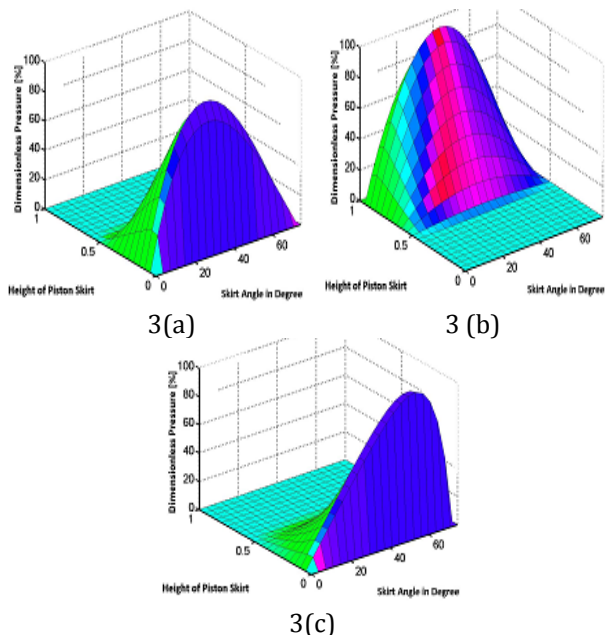
**Fig. 1.** Dimensionless eccentricities profiles, 1(a) for HD film ,1(b) for EHL film.

On the other side the contact is avoided in EHL regime due to more load carrying capacity be EHL film. Now for film thickness profiles at different crank angles are shown in Fig. 2. In Fig. 2(a) maximum and minimum hydrodynamic film profiles are shown. The difference in magnitude in both profiles is due to the application of load on lubrication film in case of minimum hydrodynamic film. The profiles show that the hydrodynamic film starts establishing from start of cycle and reaches at a peak at the end of power stroke and again goes to least value at the end of exhaust stroke. Figure 2 (b) shows profile for EHL film which is comparatively much less at initial engine start-up speeds.



**Fig. 2.** Film thickness profiles, 2(a) Hydrodynamic film, 2(b) EHL film.

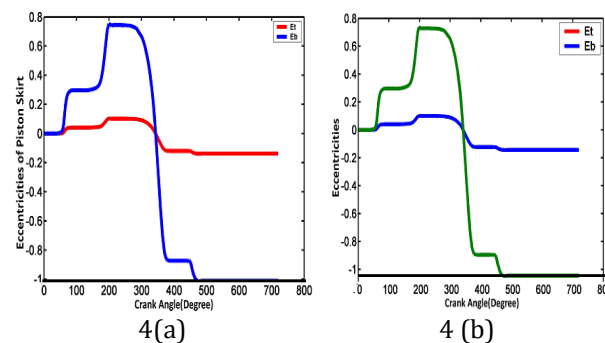
Figure 3 shows hydrodynamic pressure fields, developing on the piston skirt in three dimensions. The selected pressure fields are for crank angles of 180°, 540° and 720°, where the pressure fields are biased towards bottom of piston skirt at 180° and 720° crank angles but the pressures are shifted towards top of piston skirt in case of 540° crank angle.



**Fig. 3.** Hydrodynamic pressure fields for crank angles, 3(a) at 180°, 3(b) at 540°, 3(c) at 720°.

#### 4.2 RADIAL CLEARANCE OF 100 MICRON

As far as the eccentricities profiles are concerned, in hydrodynamic regime there is no significant change is observed by changing the radial clearance from 10 to 100 micron at 600 rpm engine start-up speed using SAE 50 oil. But in EHL regime the physical contact at lower line is shown Fig. 4(b) in case of increasing the radial clearance.



**Fig. 4.** Dimensionless eccentricities profiles, 4(a) for HD film , 4(b) for EHL film.

This shows that increasing radial clearance is not helpful in avoiding the physical contact in both regimes. In results of Fig. 5 (a) and 5(b), there is a significant increase of film thickness can be observed by increasing the radial clearance. The magnitude of film thickness profiles for both regimes shows an increase of 10 times in film thickness as the radial clearance is increased by 10 times for rough piston skirt case. Dimensionless pressure fields are also changed interestingly, in

case of 540 degrees crank angle the pressures are distributed on bottom and top of piston skirt as shown in Fig. 6(b). Similarly the magnitude of pressure is decreased at 720 degrees crank angle and the pressure field is steeper in comparison to 10 micron radial clearance results.

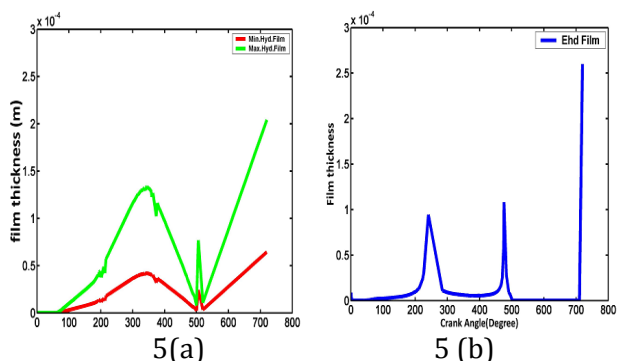


Fig. 5. Film thickness profiles, 5(a) Hydrodynamic film, 5(b) EHL film.

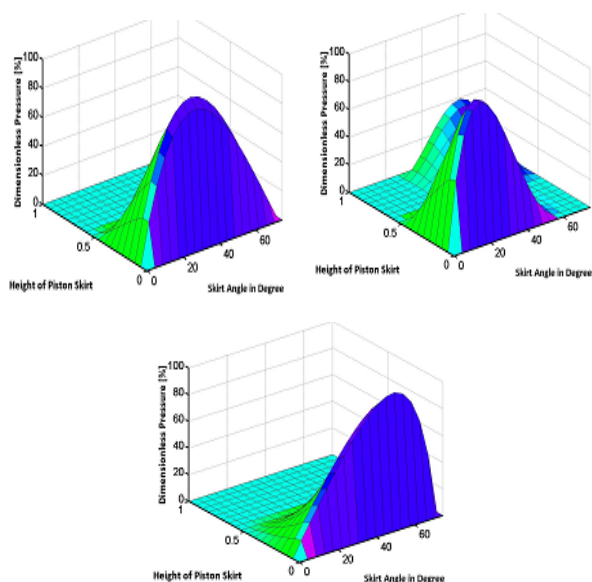


Fig. 6. Hydrodynamic pressure fields for crank angles, 6(a) at 180°, 6(b) at 540°, 6(c) at 720°.

## 5. CONCLUSION

In this work the lubrication of isotropic rough piston skirts and the liner surfaces have been considered at initial engine start-up speed of 600 rpm. The lubrication model is presented such that the contact and wear should be avoided. In the basic model the piston eccentricities reduce with the change from hydrodynamic regime to EHL. It helps in avoiding physical contact between interacting surfaces. The interesting findings of this work are the effect of radial clearance on rough piston

skirt lubrication characteristics at initial engine start-up. Increasing the piston to bore radial clearance from 10 micron to 100 micron increases the magnitude of dimensionless film thickness but the dimensionless eccentricities profiles show the contact cannot be avoided despite increase of film thickness at 100 rpm. The hydrodynamic pressure profiles for both values of radial clearance show that the pressures are pronounced at the bottom of piston skirt. To optimize the very low initial start up speed, 10 micron radial clearance should be preferred over 100 micron for improved concentricities and pressures under the stated conditions.

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