

## **A Historical Review of Gas Lubrication: From Reynolds to Active Compensations**

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Aerostatic bearing  
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### ABSTRACT

Friction and wear are tribological phenomena that have produced significant economic damage and environmental issues throughout our history. Different kinds of lubricants and coatings have been employed in the attempt of reducing friction losses. Because of their remarkable performance, gas lubricated systems are characterized by almost zero friction and wear. For this reason, they can be utilized in applications where rolling and fluid bearings cannot, e.g., metrology, medical equipment and tool machine. This paper reviews the evolution of gas lubrication through history. The topography of the subject is described from its origins to how it was shaped over the centuries, trying to preserve the logical cause and effect relationship of the events. For the sake of consistency, the review considers only the key past contributions which are strictly related to gas lubrication.

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## **1. INTRODUCTION**

Friction and wear are tribological phenomena that have produced significant economic damage and environmental issues throughout our history. The mechanical and thermal severities caused by friction along with the consequent wear are some of the most relevant problems faced by engineers in machine design. It is suffice to say that one-third of the fuel consumption of passenger cars is used to compensate frictional losses. Different kinds of lubricants and coatings are a solution currently adopted to reduce friction and avoid the presence of wear. Because of their remarkable

performance, gas lubricants make it possible to significantly reduce friction and wear, thus providing systems with high positional accuracy and low environmental impact. For this reason, gas lubricated systems are employed for high-precision applications, such as medical equipment and measuring and tool machines [1,2].

This paper reviews the history of gas lubrication. The topography of the subject is described from its origins to how it was shaped over the centuries, trying to preserve the logical cause and effect relationship of the events. For the sake of consistency, the review

considers only the key past contributions which are strictly related to gas lubrication.

The first part of the paper deals with the most important studies in fluid mechanics, i.e., the equation of motion and the equation of continuity, which made it possible to lay the foundations of lubrication theory. Secondly, the paper describes the first experiments and theories on hydrodynamic lubrication and lubricant properties. The review continues by focusing on the historical evolution of gas lubrication up to today.

## 2. THE BEGINNING: THE BIRTH OF FLUID MECHANICS

Although Daniel Bernoulli (1700-1782) was the first to publish a book in the field of fluid mechanics: *Hydrodynamica, sive De viribus et motibus fluidorum commentarii*, it is commonly recognised that the Swiss engineer Leonhard Euler (1707-1783) was the founder of the subject. Driven by his passion for mathematics, Euler was one of the most eminent scientists in history who provided the Equation of continuity and the Equations of motion for non-viscous flows. This mathematical framework was the first step in laying the foundations of fluid film lubrication theory. The next step towards the formulation of the lubrication theory was provided by the French physician Jean Léonard Marie Poiseuille (1799-1869), who analysed the properties of viscous fluids. He spent most of his life performing experimental investigations on blood flow in human capillaries. The result of these arduous experiments was a formula to study viscous fluid flowing under laminar regimes (Equation 1).

$$\Delta P = \frac{128 \mu L Q}{\pi d^4} \quad (1)$$

This equation made it possible to assess the pressure drop within laminar flow through thin pipes. He found that the pressure drop  $\Delta P$  along a straight capillary increases proportionally to the tube length  $L$ , the fluid mass flow rate  $Q$  and viscosity  $\mu$ , and it decreases with the fourth power of the conduit diameter  $d$ . Today, this formula is known as Hagen-Poiseuille's law since the same results were independently and contemporarily achieved by the German engineer Gotthilf Heinrich Ludwig Hagen (1797-

1884) during his studying on the fluid flow within brass tubes.

In the same period, Claude Louis Navier (1785-1836) along with Sir George Gabriel Stokes took advantage of Euler and Poiseuille's contributions to provide one of the milestones of fluid mechanics: Navier-Stokes Equations.

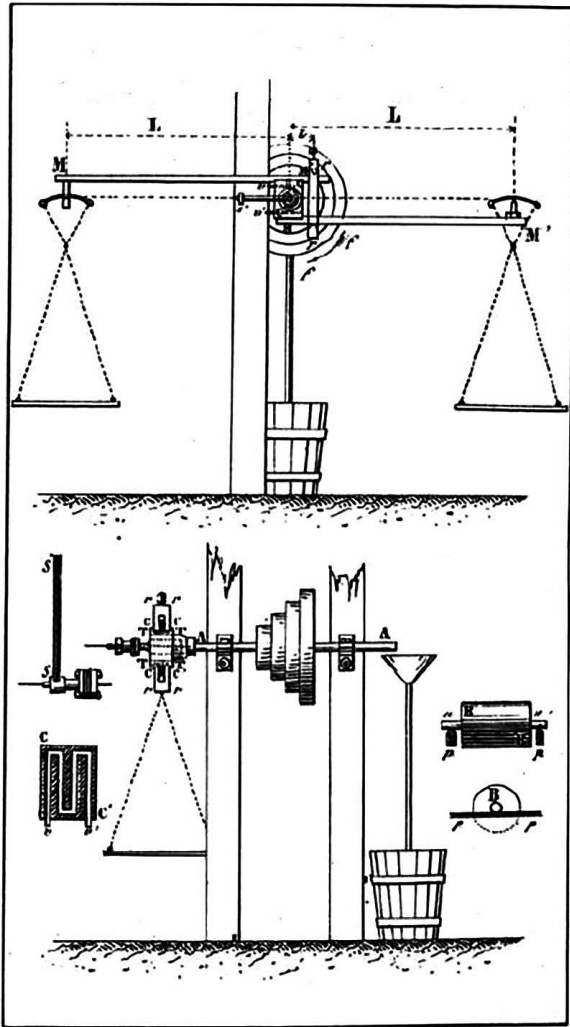
$$\begin{cases} \rho \frac{Du}{Dt} = \rho X - \frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \rho \frac{Dv}{Dt} = \rho Y - \frac{\partial P}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ \rho \frac{Dw}{Dt} = \rho Z - \frac{\partial P}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \end{cases} \quad (2)$$

This set of equations made it possible to model the motion of viscous fluids by considering the equilibrium between volume (inertial, gravitational), pressure and viscous forces.

## 3. FOUNDATION OF THE LUBRICATION THEORY: FROM EXPERIMENTS TO REYNOLDS' EQUATION

Performing experimental studies on lubricant properties was the consequential and natural step of the development of the theoretical framework by Navier and Stokes. Gustave Adolphe Hirn (1815-1890) was the first who investigated the frictional properties of different kinds of fluids. He pioneered experimental works on fluid bearings by testing twelve natural oils (both mineral and fatty oils), air and water as lubricants. To perform these experiments, he developed a fine friction balance (Fig.1).

According to Fig. 1, this friction balance consists of a half bronze bearing loaded against a polished cylindrical cast iron drum. The bearing supports a shaft that can rotate at different speeds and which is loaded with two dead weights applied at the extremities of two horizontal lever arm of the same length. Given a constant rotational speed, frictional forces were measured by adding weights to balance the misalignments between the two lever arms induced by friction. Performing these experiments allowed Hirn to discover the existence of the "frottement médiat" and the "frottement immédiat" regimes.



**Fig. 1.** Hirn's experimental apparatus for frictional measurements.

He observed that the "frottement médiate" occurs when a full fluid film separates the surfaces in the presence of relative motions, i.e., hydrodynamic regimes, whereas the "frottement immediate" takes place in dry interactions or when the fluid film lubricant is broken.

*"For water and air to act as lubricant it is necessary for the drum to turn sufficiently rapidly to drag them into the bearing. When the speed reduces to a certain value, the two non-viscous fluids are expelled by the pressure and the surfaces come into direct contact, and friction at once becomes enormous [3]"*

By experimenting with different operating conditions and lubricants, Hirn discovered that under the hydrodynamic regime, friction coefficients depend on the rotational speed, load and lubricant properties. In the presence of full film lubrication and light loads, friction is

proportional to the speed, whereas, in the presence of high loads it varies approximately according to the square root of the speed. In addition, Hirn argued that also the temperature of the lubricant may have an important role. He noted that when the system worked for a long time, friction increased the lubricant temperature and consequently induced unbalances between the torque arms. According to this, he installed a water-cooled system CC' (Fig. 1) that made it possible to keep the bearing temperature constant within plus or minus 0.1 °C, and unaware of Joule's experiments, he also provided necessary measurements to change the water temperature by 1°C (3630 J with respect to the correct value measured by Joule 4090 J [4]).

In 1879, Hirn's studies were refined by Robert Henry Thurston (1839-1903). Thurston showed that, giving an increasing speed, the friction coefficient within a fluid bearing initially decreases below its static value, and reaches its minimum and then increases to a constant value. He also noted that the value of the minimum friction coefficient varied together with the applied load.

Hirn and Thurston's research was efficiently exploited by the Russian engineer Nikolay Pavlovitch Petroff (1836-1920). Petroff laid the foundation of the hydrodynamic theory of friction and carried out further viscosity measurements with the aid of an experimental apparatus similar to the one employed by Poiseuille. His most important contribution was providing the first analytical formulation to evaluate friction coefficients in the presence of hydrodynamic lubrication. Petroff's formula correlates the friction force and bearing parameters:

$$F_v = \frac{\mu UA}{\varepsilon + \frac{\mu}{\lambda_1} + \frac{\mu}{\lambda_2}} \quad (3)$$

where,  $\varepsilon$  and  $\mu$  are the film thickness and viscosity of the lubricant,  $U$  is the tangential relative velocity between the interacting surfaces and  $A$  is the contact area. The coefficients  $\lambda_1$  and  $\lambda_2$  consider slip phenomena at walls of the interacting surfaces. Despite much effort devoted to performing studies on hydrodynamic lubrication, Petroff was not able to find the relation that exists between friction and the load carrying capacity of a fluid bearing.

The first scientist who explained a principle that endows fluid bearings with their load carrying capacity was Beauchamp Tower (1845-1904) in 1883-1884. Tower was appointed by the Committee of the Institution of Mechanical Engineering to investigate on how speed, the material of surfaces and lubricant properties may affect the friction within rotating shafts. To this regard, he developed a test rig to measure the friction of fluid lubricated bearings (Fig. 2).

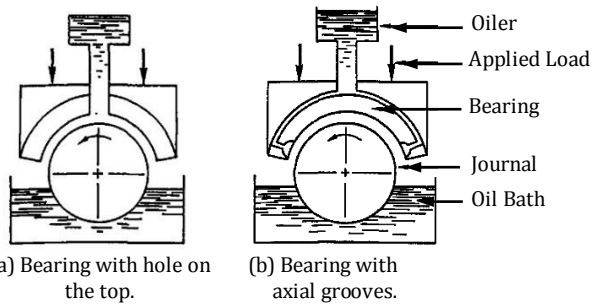


Fig. 2. Tower's experimental bearings (from [5]).

Tower's apparatus consisted of a gun-metal half bearing that supported a steel journal. He drilled a hole on the top surface of the bearing to supply it with lubricant from an oiler bath, as shown in Fig. 2a. However, after some experiments, Tower was aware that this configuration did not allow tests to be performed, as when the journal rotated, a considerable amount of oil was pumped out from the bearing. Hence, he inserted a wooden plug into the supply hole to stop leakages. Despite this modification, the solution was not effective to stop the leakage because the lubricant was pumped out anyway after the plug was expelled. To obtain a better understanding of this phenomenon, he substituted the wooden plug with a manometer detecting pressure peaks up to 1.4 MPa: the existence of a hydrodynamic pressure was documented for the first time. After this evidence, Tower concluded that the shaft rotation pressurized the oil film, thus providing the bearing with a load carrying capacity. Following his insight, he suitably modified the configuration of the bearing feeding system and demonstrated that, to achieve good performance, the oil had to be supplied from unloaded areas, allowing it to flow into the high-pressure zone. Additionally, Tower obtained the first map of hydrodynamic pressure (Fig. 3) by using several pressure gauges that were circumferentially installed on the bearing surface.

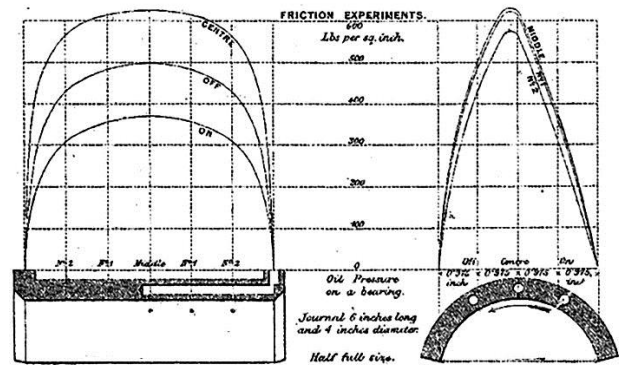


Fig. 3. Tower's measurements of the pressure developed in a journal bearing (from [6]).

However, Tower was not able to explain theoretically the formation of the pressure gradient within the lubricant film.

Just a few years later, in 1886, Professor Osborne Reynolds reviewed Tower's reports and explained these experimental findings by utilising a rigorous theory. He combined a simplified form of Navier-Stokes equations (Equation 1) with continuity equation (Equation 4) and obtained a set of partial differential equations to compute the pressure distribution within lubricants. By considering a Cartesian reference frame where  $x$  and  $z$  are the axis identifying planes parallel to the bearing surface and  $y$  the direction along which the lubricant film thickness  $h$  is measured.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\begin{cases} \frac{\partial P}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial P}{\partial y} = 0 \\ \frac{\partial P}{\partial z} = \mu \frac{\partial^2 w}{\partial y^2} \end{cases} \quad (4)$$

where,  $P$  and  $\mu$  are the pressure and viscosity characterising the lubricant film, whereas  $u$ ,  $v$  and  $w$  are the velocities respectively along  $x$ ,  $y$  and  $z$  directions. Equation 4 was notably simplified by considering the fact that fluid film thicknesses are small compared to the bearing dimensions. Indeed, this consideration makes it possible to neglect weight and inertial terms because they are small compared to the viscous ones. Finally, by considering the case of incompressible lubricant (oils) and the following boundary conditions:

$$y = 0 \quad u = U_0 \quad w = 0 \quad v = 0$$

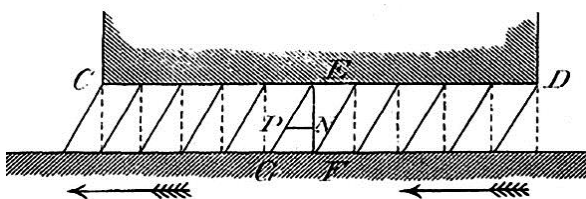
$$y = h \quad u = U_1 \quad w = 0 \quad v = U_1 \frac{\partial h}{\partial x} V_1$$

he obtained the equation that today is known as Reynolds' Equation:

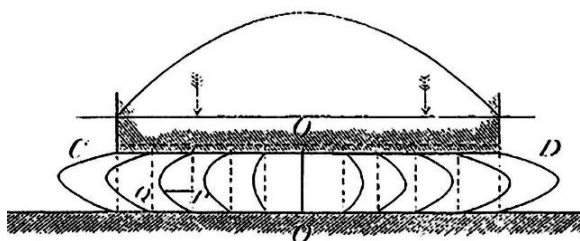
$$\frac{\partial}{\partial x} \left( h^3 \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial P}{\partial z} \right) = 6\mu \left[ (U_0 + U_1) \cdot \frac{\partial h}{\partial x} + 2 V_1 \right] \quad (5)$$

Reynolds' mathematical framework made it possible to identify the phenomena which generate the bearing carrying capacity, i.e., Poiseuille, Couette and squeeze effects. The Poiseuille term describes the net flow rates due to the pressure gradient within the lubricated area, whereas the Couette and the Squeeze terms describe respectively the net entering flow rates due to the tangential and normal velocities of the interacting surfaces. Moreover, Reynolds provided a qualitative description of the pressure distributions that are established under different operating conditions (Fig. 4):

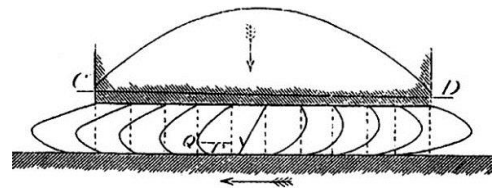
1. Parallel surfaces in relative tangential motion (Fig. 4a)
2. Parallel surfaces approaching with no tangential motion (Fig. 4b)
3. Parallel surfaces approaching with tangential motion (Fig. 4c)
4. Surfaces inclined with tangential movement only (Fig. 4d)
5. Revolving motion (Fig. 4e)



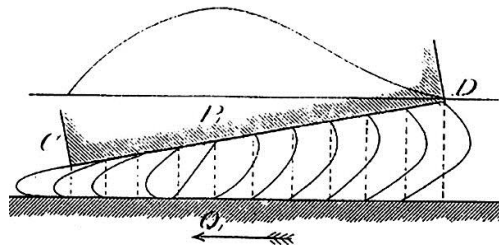
(a) Parallel surfaces in relative tangential motion.



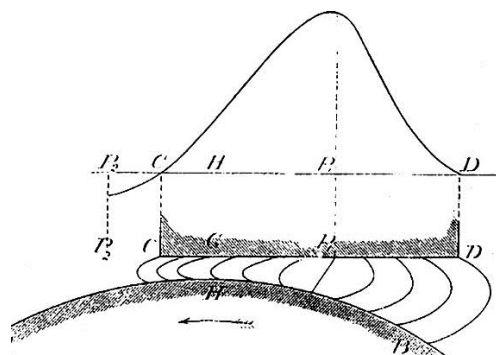
(b) Parallel surfaces with no tangential motion.



(c) Parallel surfaces with tangential motion.



(d) Surfaces inclined with relative tangential motion.



(e) Revolving motion.

**Fig. 4.** Different cases analysed by Reynolds.

Along with this mathematical construction, Reynolds also presented some experiments where he showed that olive oil viscosity changes with an exponential law with respect to the temperature.

$$\mu = \mu_0 e^{-\alpha(T-T_0)} \quad (6)$$

where,  $T_0$  is a reference temperature and,  $\mu_0$  and  $\alpha$  are two constants.

#### 4. GAS LUBRICATION AN ACCIDENTAL DISCOVERY

Reynolds' works laid the foundations of hydrodynamic theory. In addition, his theory developed gas lubrication theory and many other fields of study. The rest of this paper deals specifically with the history of gas lubrication and does not consider the other fields of studies which fall outside the aims of this work.

Although Hirn was the first to use air as a lubricant, the American professor Albert

Kingsbury (1863-1943) is considered as the father of gas lubrication, in the light of his valuable contribution in the field. Kingsbury accidentally discovered that gas films exhibit load carrying capacity while he was performing frictional measurements on his screw threads device (Fig. 5).

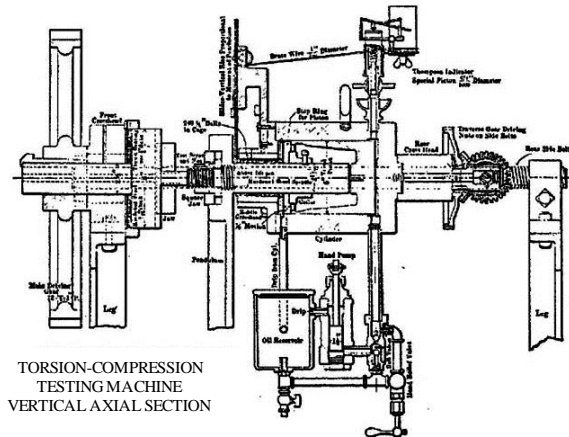


Fig. 5. Kingsbury's device for measuring friction in screw threads (from [6]).

He was employing a test bench consisting of a vertical piston deliberately mounted without seals in the attempt to reduce friction. To compensate the absence of sealing, the piston had a small interference with respect to the cylinder which it slides within it, allowing the formation of a thin air gap layer between the cylinder and the piston. Kingsbury noticed that when the piston received a whirl around its longitudinal axis it could rotate continuously for several minutes without touching the cylinder surface.

Fascinated by this evidence, he built another test rig where the piston was placed horizontally, and the same test was repeated, obtaining the same result: he designed the first air bearing in history.

Once he made sure that pressurised air gaps could bear loads, he designed a fine air bearing device (Fig. 6) which presented a piston heavier than the previous one and a cylinder with several circumferential holes to make it possible to measure the air gap pressure through mercurial manometers (Fig. 7). The outer surface of the piston and the inner of the cylinder were carefully grinded and the diameters of these two cylindrical surfaces differed by 0.0406 mm.

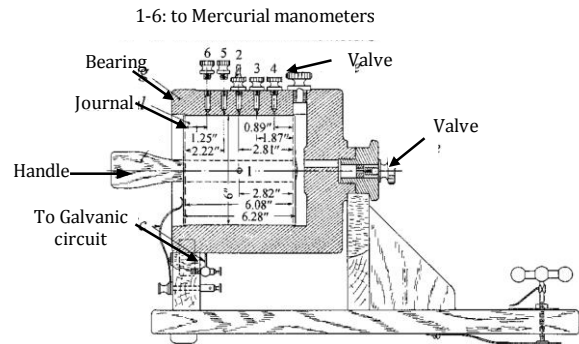
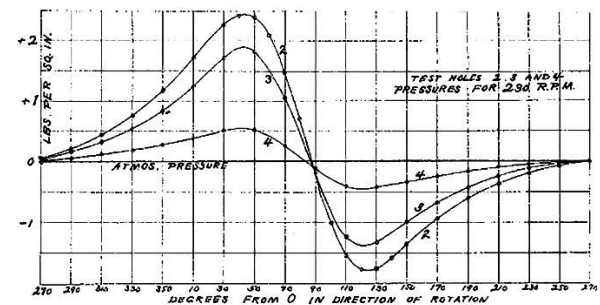
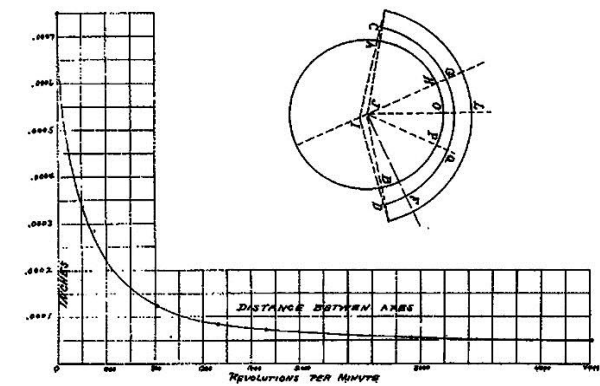


Fig. 6. Kingsbury's air bearing device (from [7]).



(a) Air gap pressure vs degrees of rotation



(b) Air gap height vs revolutions per minutes

Fig. 7. Results from Kingsbury's experiments on his air bearing device (from [6]).

The piston rotation was set manually acting on a wooden handle H and when higher speeds were necessary, an ordinary geared hand drill was used. A galvanic cell with a signal bell were used to detect the presence/absence of contact between piston and cylinder. To measure the air gap height, the galvanic cell was used in combination with small screws placed at different locations around the cylinder circumference.

With the aid of this apparatus, Kingsbury observed that at low speeds the piston surface rolled on the cylinder, whereas when the speed was gradually increased, the air gap pressure allowed firstly the lifting of the piston and, secondly the alignment of its axis with that of

the cylinder. In addition, Kingsbury verified that at sufficiently high speed the air was an optimum lubricant exhibiting a friction coefficient of about  $10^{-5}$ .

Investigating Reynolds' work and observing his results, Kingsbury recognized the existence of a center pressure and exploited this insight to conceive the first tilting pad in history (1898). This bearing presented surfaces pivoted around the center of pressure so that due to the antisymmetric shape of their pressure distribution, they can automatically provide a convergent gap, thus increasing the carrying capacity (Figure 8). This was one of the finest bearings that have ever built. Despite his startling insight, the paternity of this invention could not be attributed to him until 1910.

When he applied for a patent in 1907, his application was rejected because, unbeknownst to him, the Australian engineer Anthony George Maldon Michell had already filed a similar patent in 1905 (Fig. 9). Other than this patent, Michell provided important theoretical contributions by solving Reynolds' equation for the case of an inclined rectangular block which slides over a plane surface with finite width.

The novelty of this solution stemmed from the fact that, being a three-dimensional solution, it was possible for the first time to evaluate the effects of side leakage [8].

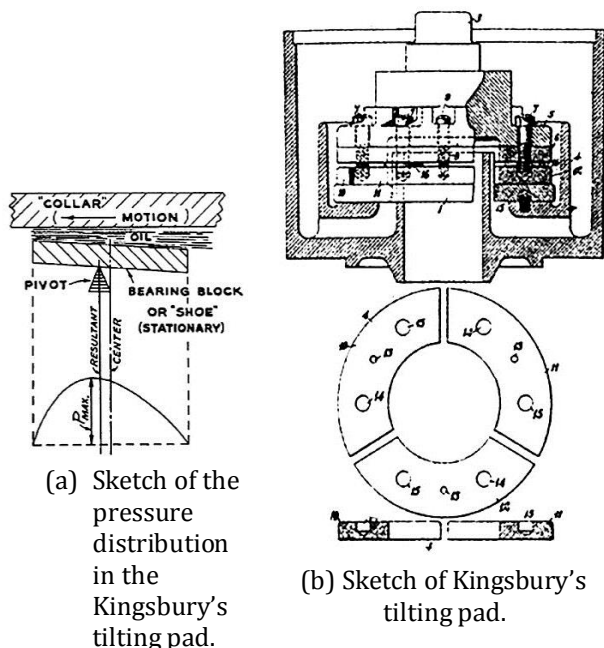


Fig. 8. Kingsbury's tilting pad (from [6]).

As a skilled analyst, Michell expressed the pressure in the form of an odd Fourier series:

$$p(x, z) = \sum_{n=1,3,5}^{\infty} \frac{f(x) \sin(nz)}{nx} \quad (7)$$

where,  $f(x)$  is a Bessel function whose coefficients are null at the edge of the slider. Further details on this kind of approximations can be found in [9]. This solution represented a useful design tool that also contributed to Michell's career as a businessman.

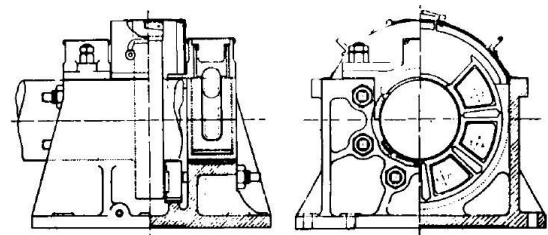


Fig. 9. Michell's thrust bearing, used in the early merchant marine installation (from [8]).

After Michell, many of the following studies consisted of trying to solve Reynolds' equation (Equation 7) in presence of further hypotheses and different boundary conditions. Among those who solved Reynolds' equation, Arnold Johannes Wilhelm Sommerfeld occupied a relevant position. Apart from his notable scientific contributions to many fields such as atomic structures, quantum theory, spectral analysis and the theory of relativity, Sommerfeld also provided a significant contribution in the field of hydrodynamic lubrication. He succeeded in the integration of  $\frac{d\theta}{(1+\epsilon \cos(\theta))^3}$ , which Reynolds had failed to obtain, by using a simple substitution:

$$(1 + \epsilon \cos(\theta)) = \frac{1 - \epsilon^2}{(1 - \epsilon \cos(\psi))} \quad (8)$$

and applying periodicity as boundary conditions.

$$p(0) = p(2\pi)$$

$$\left. \frac{dP}{d\theta} \right|_0 = \left. \frac{dP}{d\theta} \right|_{2\pi} \quad (9)$$

Sommerfeld obtained an explicit analytical expression for pressure distribution, load locus of shaft center and friction. These contributions were used by W.J. Harrison for deriving the basic equation of gas lubrication that today is known as Harrison's Equations. Rather than considering a constant density, Harrison combined Reynolds' and gas equations under the assumption of

isothermal conditions, and obtained a more general mathematical formulation:

$$\frac{\partial}{\partial x} \left( \frac{h^3}{\mu} \frac{\partial P^2}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial P^2}{\partial z} \right) = 12 U \frac{\partial(P_h)}{\partial x} \quad (10)$$

As in the case of Tower and Reynolds, Kingsbury's insight also received a theoretical foundation. However, due to its non-linearity, Harrison was not able to obtain a closed form analytical solution of his formulation. Because of this, he achieved an approximated solution by manually applying Runge-Kutta's method. Figure 10 compares the numerical results achieved by Harrison with the experimental ones measured by Kingsbury.

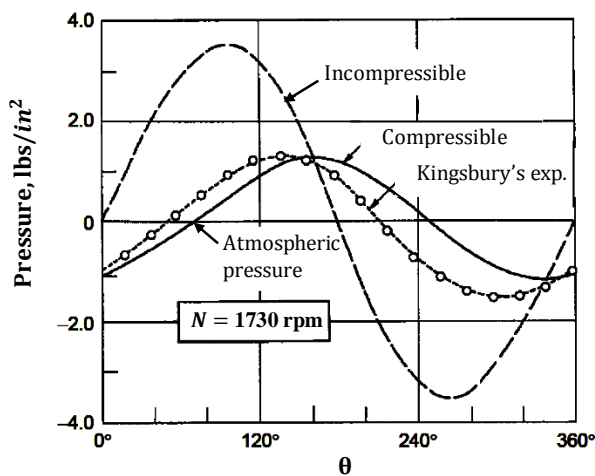


Fig. 10. Comparison between the numerical and experimental results respectively obtained by Harrison and Kingsbury (from [7]).

Another invention worthy of note is the thrust bearing designed by Stone in 1921. The innovation provided by Stone was an air bearing with a glass thrust collar and a quartz shoe. The introduction of these two components made it possible to provide a measurement of the air gap thickness via optical interferometer bands for the first time. From the mid-twenties further studies were regarded to the conceptualization of the bearing dynamics and stability.

In 1925, Stodola and Newkirk developed a valuable study on the oil whip phenomenon. They described this phenomenon as a vibration that may occur when shafts spin above their critical speed.

*“Whipping in general may be defined as a resonant vibration or whirling of the shaft of a machine when running at other than the resonant or critical speed”.*

Consequently, they stated that this form of vibration was strictly related to the stiffness and damping of bearings and an understanding of this dynamic phenomenon cannot be obtained without considering these aspects. This new insight rised a new field of research that today is known as rotordynamics.

Figure 11 outlines the historical perspective provided herein up to 1925.

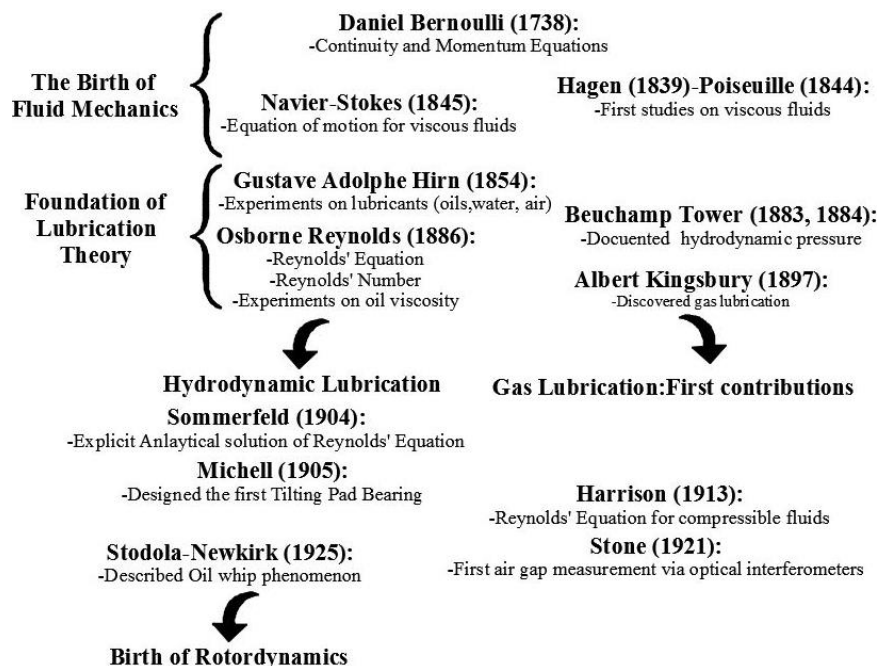


Fig. 11. Historical perspective regarding lubrication theory and fluid mechanics between 1738 and 1925.



## 5. THE DOLDRUMS: THE SECOND WORLD WAR

Unlike the first part of the twentieth century, the period between the end of the First and the Second World War was a period of doldrums regards to the theory of lubrication. In these years of scientific depression, Swift's work was the only one worthy of note. He rearranged Reynolds' equation to extend it to cases where dynamic loads are applied to bearings. Through this new arrangement of the Reynolds' equation, it was possible to carry out the first stability analysis of that time. Swift used his knowledge on the subject to provide one of the first optimal design procedures for fluid bearings.

## 6. THE DAWN-AGE: GAS LUBRICATION AS AN INDEPENDENT

The doldrums between the two World Wars were followed by a very prosperous period as far about technical innovations and inventions. In fact, the fifties may be probably considered the most flourishing period in the history of gas lubrication theory up to today. A large interest in using gas bearings arose because their inherent features were perfectly in accordance with the requirements of many industrial purposes of the time. Thanks to the low influence of temperature variations on air properties, aerostatic bearings were successfully employed for different applications. The small viscosity changes over a wide range of temperature (from -267 to 1648 °C), low tendency to change in phase and exhibit cavitation all made it possible to use air bearing in high-temperature missiles and low-temperature cryogenic applications. During this dawn-age, gas bearings were also installed in spacecraft gyroscopes and nuclear reactors in United States and England respectively. In addition, unfortunately, they were widely employed for nuclear power, thus contributing to the East-West cold war.

This widespread appeal and the diversity of applications made it possible to consider gas lubrication as a separate and independent field with respect to fluid lubrication. Fluid bearings were utilised in applications characterised by small temperature variations, where large load carrying capacities were required. Conversely, because of their low changes in viscosity with respect to temperature variations and their

almost zero friction and wear, gas bearings were successfully used in high speed spindles and metrology. From a historical point of view, the birth of the field of gas technology as an independent field can be dated as 1959, when the First International Symposium on the field was held in Washington. In this congress, various activities and the future directions of gas lubrication technology were defined.

## 7. PROGRESS OF THE GAS LUBRICATION TECHNOLOGY

The bar chart shown in Figure 12 illustrates the number of papers written on gas lubrication between 1950 and 2000 in *Transaction ASME*.

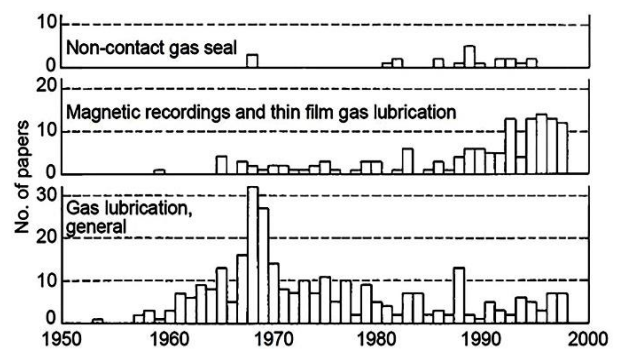


Fig. 12. Number of paper written on gas lubrication (from [7]).

This chart makes it possible to understand the evolution of gas lubrication technology during the second part of the twentieth century. The first significant scientific publications appeared in the sixties after the first International Symposium on gas lubricated bearings. These years witnessed the invention of the first porous bearing, which was designed and developed in the United States National Labs. The novelty of this type of bearing stemmed from its exhaust system consisting of a porous surface. Porous surfaces made it possible to obtain more regular air gap pressure distribution and higher stiffness, stability and load carrying capacity. Further details on this kind of air bearing are provided by Montgomery, Djordjević et al. [10] and Fourka et al. [11]. However, at the beginning, porous technology was not widely employed due to the lack of accuracy of the manufacturing process of the time. In the same period, air bearings along with the use of granite counter surfaces were exploited to develop the first Coordinate Measuring Machines (CMM) in 1965.

Another thriving force behind the boom in gas lubrication was the increasingly demand for ultra-precise supports and positioning systems to produce large hard disk drives and semiconductor parts. The small air bearing gaps brought significant advance in magnetic recording.

Regarding bearing design and modelling, the advent of modern digital computers and improvements in manufacturing technology allowed empirical formulas and analytical formulations to be replenished by more reliable and accurate numerical models. The use of numerical methods permitted more accurate approximate solutions of Reynolds' equation, to be achieved, thus providing better predictions of bearing behaviour. These numerical studies made it possible to improve bearing design and perform accurate stability analysis.

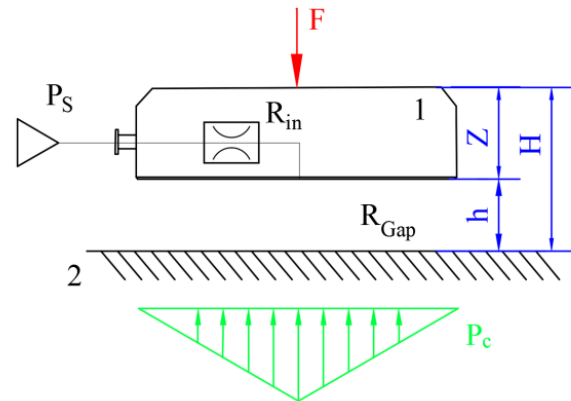
In 1957 Roudebush developed one of the first investigations on the influence of geometrical parameters and temperature over bearing performance. Several linearized models were proposed in order to study the dynamic performance of both journal and thrust bearings.

As it is evident in Fig. 12, the number of papers on gas lubrication significantly increased in the period between the sixties and seventies due to the birth of the *Journal of Lubrication Technology*, reaching its peak during the period in which the Second International Conference on Gas Lubrication was held (1968). In these years, Gross (*"Gas film lubrication"*) and Powell (*"Design of aerostatic bearings"*) provided two of the most important milestones of gas bearing design.

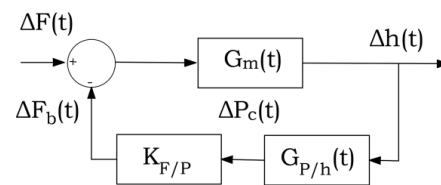
Moreover, the increasing demand for accuracy and the constant enhancement in the field of manufacturing led to new design methodologies and theoretical approaches. Novel theoretical studies allowed more accurate predictions of air bearing performance to be achieved. Many of these investigations were reviewed in a valuable work by Castelli and Pirvics.

The advent of control theory was a spur for Donald and Wilcock, who employed a new stand point to study bearing behaviour. They considered bearings as closed loop servo-systems. To clarify this line of thought, it is worth analysing Fig. 13, where a sketch of a conventional pad and the associated block

diagram are shown respectively in Figs. 13a and 13b. Figure 13a illustrates the scheme of a conventional aerostatic thrust bearing, where a thin air gap layer separates the floating bearing 1 and the stationary part 2.



(a) Schematic representation of an aerostatic thrust bearing.



(b) Closed-loop servo model of a conventional gas bearing.

**Fig. 13.** Sketch of a conventional pad and the associated block diagram.

Here, air is pressurised up to the supply pressure  $P_s$  by a compressor and secondly it passes through the bearing before being exhausted by its restrictors. Once in the clearance, the air flows through the bearing land where it gradually reduces its pressure from  $P_c$ , close to the restrictor area, to the atmospheric one  $P_{Amb}$  at the outer edge of the bearing. This pressure distribution, which can be roughly estimated as a mean pressure  $P_m$  multiplied by the bearing land, makes it possible to balance the load applied upon the bearing  $F$ . When the bearing is subjected to increasing (decreasing) load variations the air gap reduces, providing an increase (decrease) of the mean pressure that permits the compensation of the load variations. In the light of their operating principle, bearings were studied through closed loop block diagrams. An example of these block diagrams is shown in Fig. 13b. Here, all functions and variables are expressed as functions of time  $t$ . This block diagram consists of a direct and a feedback branches. The direct branch comprises

the inertial transfer function  $G_m(t)$ , which embodies the masses of the bearing and the supported structure. While, the whole feedback branch represents the air gap transfer function, which is given by the product of the transfer function  $G_{P/h}(t)$ , regarding the stiffness and damping capabilities of the bearing, and the constant  $K_{F/P}$  [12–14]. The external force variation  $\Delta F(t)$  is the input to the bearing that produces the air gap variation  $\Delta h(t)$  as output. The relation between input and output depends on the inertial transfer function  $G_m(t)$ . Equilibrium conditions are reached when the difference between the external force  $\Delta F(t)$  and the bearing force variation  $\Delta F_b(t)$  is null. The variation of the bearing force  $\Delta F_b(t)$  is given by the product of the air gap pressure variation  $\Delta P_c$  multiplied by the bearing surface (the constant  $\Delta K_{F/P}$ ).

In the light of this new insight, many of further theoretical studies utilised Bode diagrams and the Nyquist's stability criterion to investigate on bearing dynamic stability.

## 8. THE USE OF THE FIRST COMPENSATION METHODS

The wide knowledge gained on gas lubrication until that time (1960-1970) and the increasing demand for high performance encouraged the experts in the field to test novel strategies aimed to raise the limits of bearing performance. To achieve this goal, the first compensation strategies were developed. The aim of these methods was to enhance the bearing performance by suitably modifying their feeding system and structure. These methodologies can be broken down in two main categories: passive and active compensation. Passive compensation strategies make it possible to improve the bearing performance by using only passive devices, e.g., valves, elastic orifice, which exploit only the energy resulting from the bearing supply air. Conversely, active compensation is achieved by employing active elements as actuators, sensors and controllers that require external sources of energy to work.

### 8.1 Passive compensation methods

Passive compensation strategies were the first adopted compensation methods because of their reduced complexity and ease of integration.

Most of the passive compensation methods used up to today, employ movable orifices, compliant elements, diaphragm valves and disk-spring damper compensators to compensate for load variations.

Using elastic orifices was one of the first passive compensation strategies, employed by Newgard et al. in 1966. Elastic orifices, thanks to their elastic compliance, made it possible to suitably adjust the air flow exhausted from the bearing, and compensating the load variations.

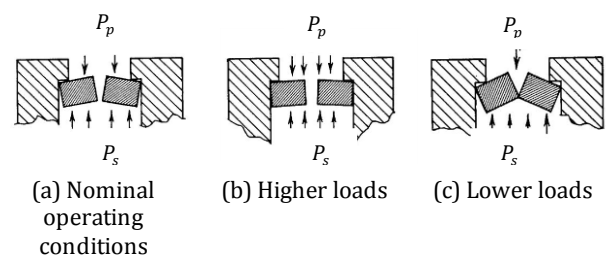


Fig. 14. Working principle of an elastic orifice.

Figure 14a illustrates the arrangement of an elastic orifice working under standard operating conditions. As it is possible to see, the orifice is embedded into the bearing in such a way that its diameter can enlarge up to its maximum dimension when the air gap pressure increases (Fig. 14b). On the contrary, when the external load decreases, the initial air gap height is restored because the lower air gap pressure deforms the orifice producing reductions of the air flow exhausted from the bearing.

The operating principle of this solution was inspiring for many further authors who exploited the deformation of elastic members to develop further compensation strategies. During the following International Gas Bearing Symposiums (the fifth, sixth and seventh), Kilmister [15], Rowe et al. [16] along with Lowe [17] proposed the first bearings with compliant surfaces, whereas Blondeel et al. [18] provided a procedure for their design calculations.

Figure 15 illustrates a scheme of the aerostatic thrust bearing patented by Blondeel [19]. In this technical solution, load compensations were achieved exploiting a compliant convergent gap geometry.

The compliant surface of the bearing deflects according to the pressure difference between the supply chamber and the air gap of the bearing.

When the external load increases (decreases), by increasing (decreasing) the air gap conicity, the air gap pressure increases (decreases) too. This conicity change makes it possible to increase (decrease) the air gap pressure by reducing the related air gap height variation.

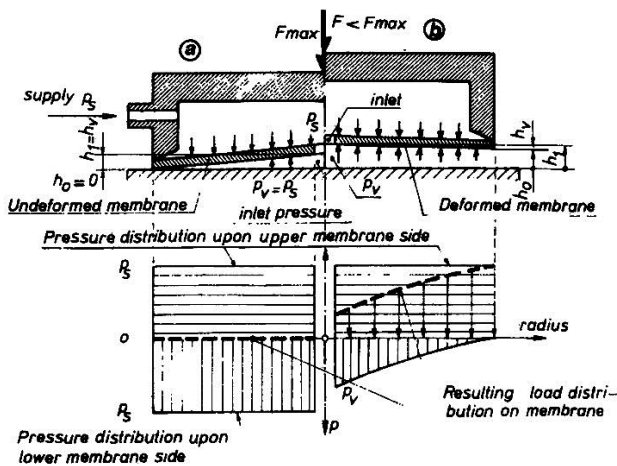
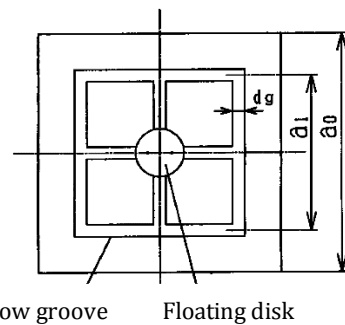


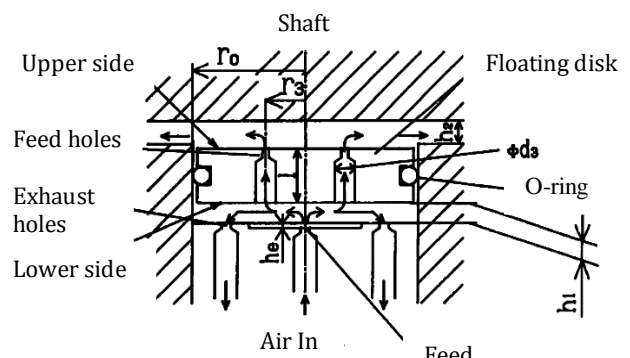
Fig. 15. Aerostatic thrust bearing with a compliant convergent gap geometry (from [12]).

In journal bearings, membrane type variable flow restrictors were used as compensating elements. In 1974, Cusano [20] proposed an externally pressurised journal bearing that employed elastic membranes to vary bearing exhausting flow, and compensating load variations. At the beginning of the nineties, Mizumoto [21] and Yoshimoto et al. [22] were the first to employ movable restrictors to compensate load variations. About this kind of restrictors, Fig. 16 presents a scheme of the bearing with a floating disk developed by Yoshimoto. Once the air is supplied, it reaches the air gap  $h_1$  passing through the hole  $d_1$  and from there it can be exhausted to the environment by the restrictor  $d_2$ , or sent into the bearing gap  $h_2$  through the hole  $d_0$ . In the presence of load variations, the floating disk moves in such a way to restore the initial bearing position by controlling the air flow that passes through the bearing. For example, when the external load increases, the air gap height  $h_2$  reduces, producing an increase of the pressure that moves the floating disk downward.

Consequently, the air gap  $h_1$  and the mass flow through the hole  $d_2$  are reduced too, increasing the pressure upon the lower surface of the disk and restoring the initial value of the air gap height  $h_2$ .



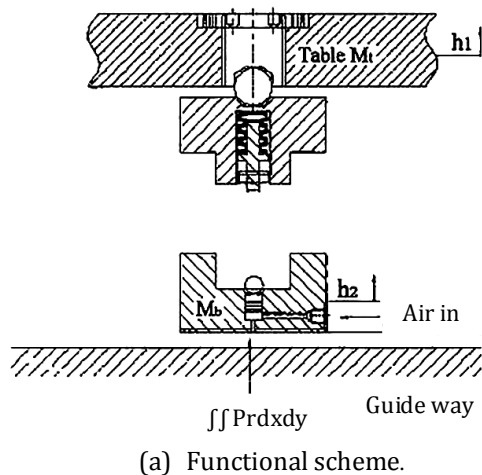
(a) Aerostatic thrust bearing with a self-controlled restrictor employing a floating disk.



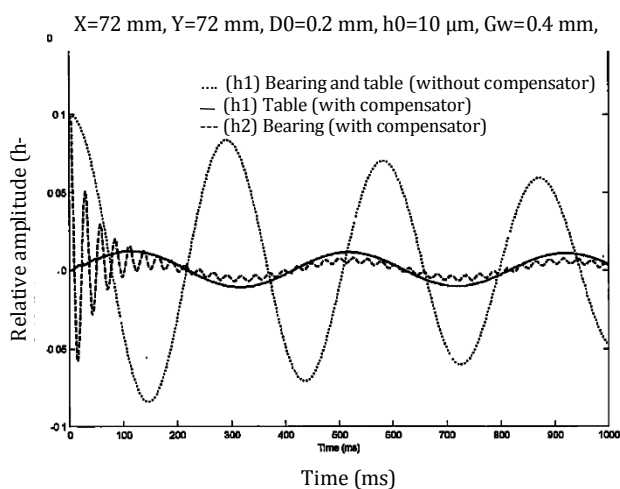
(b) Details of the self-controlled restrictor.

Fig. 16. The thrust bearing with a self-controlled restrictor proposed by Yoshimoto (from [22]).

The last passive compensation solution presented herein is by Chen et al. [23], who designed an X-shaped grooved bearing with an embedded disk spring damper compensator. As it is possible to see from Fig. 17, the bearing consists of two parts that are connected via a spring damper compensator. The presented numerical and experimental studies that were carried out demonstrated that the compensator provided significant performance improvements. Particularly, in the presence of the compensator, the bearing exhibited shorter transient and lower amplitude of oscillation (Fig. 17b).



(a) Functional scheme.



(b) Experimental results.

**Fig. 17.** The aerostatic thrust bearing with a disk spring damper designed by Chen et al. (from [23]).

Although passive compensation methods are cheap and relatively simple solutions, they are characterised by limited performance. In fact, these methodologies may not generally provide quasi-static infinite stiffness<sup>1</sup>. Moreover, when quasi-static stiffness is achieved, this is limited to relative small load variations (up to about 30% of the nominal load) and over a fraction of the bearing operating range (up to 20 % of the bearing load range) [1,13].

## 8.2 Active compensation methods

Because of limitations characterizing passive compensation solutions, active compensation methods were introduced. As discussed before, these methods consist in using active devices as

<sup>1</sup> Quasi-static infinite stiffness means that the bearing can keep constant its position (around an operating point) after the application of load variations provided at low frequencies.

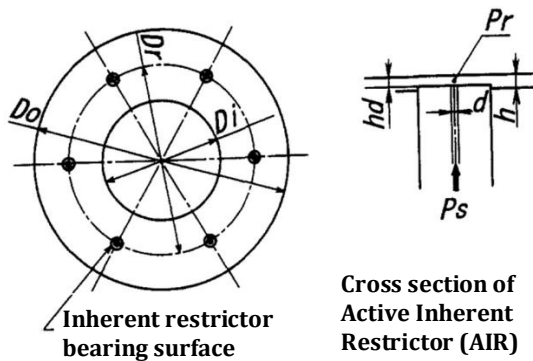
actuators, sensors and controllers to obtain actively compensated systems. Many type of actuators and sensors were adopted in actively compensated bearings.

Most of these solutions exploit piezo-actuators because of their higher dynamic, power density and efficiency. However, also magnetostrictive, electromagnetic and pneumatic actuators were successfully adopted. Regarding the sensor employed to obtain feedback on the bearing operating position, capacitive, eddy current and optical sensors are usually integrated. For further details, Raparelli et al. [1] provided a valuable critical review on actively compensated aerostatic thrust bearings. Up to today, active compensation methods can be classified in three main categories [1,24]:

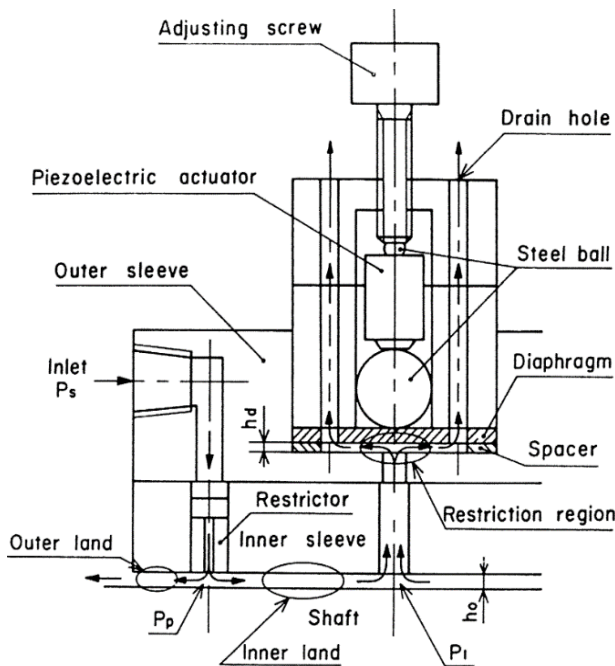
1. Active flow resistance compensation methods,
2. Active geometrical compensation methods,
3. Hybrid active compensation methods.

*Active flow resistance compensation* makes it possible to control the bearing behaviour with the aid of orifices that suitably modify the air flow exhausted by bearing restrictors. Controlling the opening of restrictors makes it possible to adjust the air gap pressure distribution, and keeping unaltered the nominal operating position of the bearing. Depending on the position of the embedded active restrictors, it is possible to distinguish among *upstream*, *air gap* and *exhausting restrictors*. These kinds of active compensation methods were employed both in journal and thrust bearings. Mizumoto et al. designed an active inherent restrictor (AIR) [25] (Fig. 18a) and an exhaust control restrictor (ECR) [26] (Fig. 18b), achieving spindles with both radial and axial infinite quasi-static stiffness. Furthermore, the use of AIRs as exhausted restrictors, made it possible to provide the spindle with a bandwidth of 50 Hz.

Morosi and Santos [27] used upstream active restrictors to control the radial injection system of an hybrid gas lubricated bearing. This restrictor consisted of a piezo-actuators and a Belleville washer that are used to control the stroke of a pins that, in turn, modifies the air supplied into the bearing clearance (Fig. 19). This active system was modelled by using a modified form of the compressible Reynolds equation [28], which takes into account the presence of the described radial injection system.

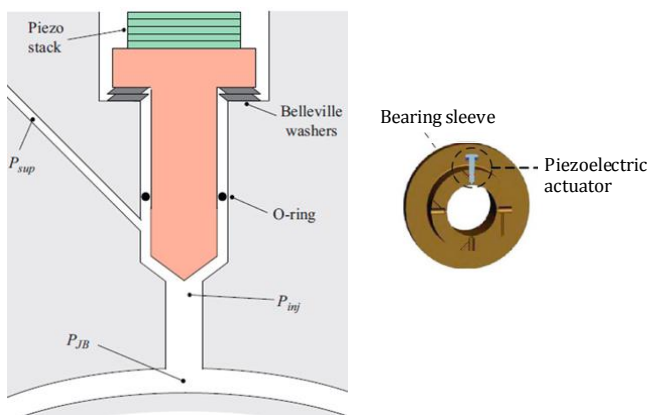


(a) Active Inherent Restrictor (AIR)



(b) Exhaust controlled restrictor (ECR)

**Fig. 18.** The two active compensation method provided by Mizumoto et al. (from [26]).



(a) The injection system designed by Santos et al.

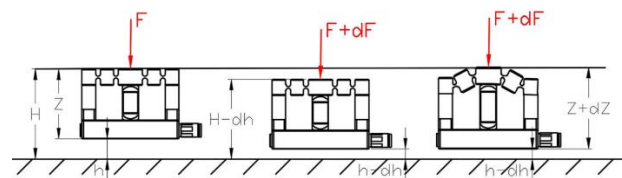
(b) 3D view of the injection system

**Fig. 19.** Active compensation solution proposed by Santos et al.(from [27]).

Once improved [29–31], it was theoretically and experimentally demonstrated that this active compensation solution makes it possible to extend the use of gas journal bearings for supporting flexible rotors. Before this remarkable achievement, conventional gas journal bearings were not commonly employed for this kind of application because of their low damping. When rotors cross their critical speeds, the low damping of the air film leads to unacceptable levels of vibrations which may significantly compromise performance and even the structural integrity of the system. Despite their lower dynamics and higher dimensions, also magnetostrictive [32], pneumatic [33] and electromagnetic [34] actuators make it possible to improve bearing performance.

*Active geometrical compensation methods* provide the possibility to obtain bearings with higher performance by changing the bearings geometry. Despite its effectiveness, this way of compensation was one of the less common methods as it was more expensive in comparison with other solutions. Inspired by the work of Matsumoto, H., et al. [35], Colombo et al. [36–37] employed this kind of compensation, by integrating a conventional pad with a compliant mechanism and a piezo actuator.

1) Initial condition    2) Perturbation    3) Compensation

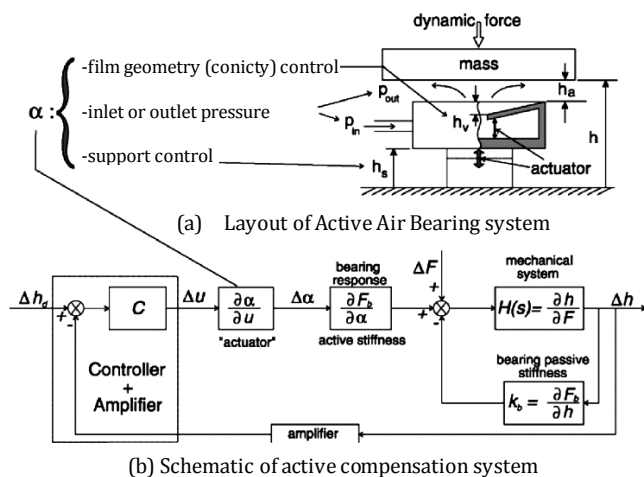


**Fig. 20.** The operating principle of the support compensation method used by Colombo et al. (from [36]).

Figure 20 illustrates the operating principle of Colombo et al. prototype. The dimension  $H$  is the controlled height (that corresponds to the bearing position). This dimension is given by the sum of the air gap height  $h$  and the vertical dimension of the pad  $Z$ . To guarantee the accuracy and precision of positioning, the system must keep its position unaltered by quickly compensating the positional variations. In the presence of constant supply pressure, when the load  $F$  increases (decreases) by  $dF$ , the controlled height decreases (increases) by  $dH$  due to the consequent air gap compression

(expansion) and mechanical deformations. In these circumstances, displacement capacitive sensors (not shown) perceive this variation and communicate with a PI digital controller that suitably supplies a driving voltage at the piezo actuator terminals. This voltage modifies the actuator stroke that makes it possible to restore the initial position of the pad by deforming the compliant mechanism.

*Hybrid active compensation methods* are a combination of the two previously discussed methods. According to author knowledge, the main contributors in designing such a hybrid systems were Al-Bender, Aguirre and Van Brussel from *KU Leuven University* [38–40]. They developed a convergent gap thrust bearing where air gap conicity and supply pressure can be simultaneously modified via piezo actuators, displacement capacitive sensors and a PID controller. Figure 21 illustrates a sketch of this active solution and the related block diagram.



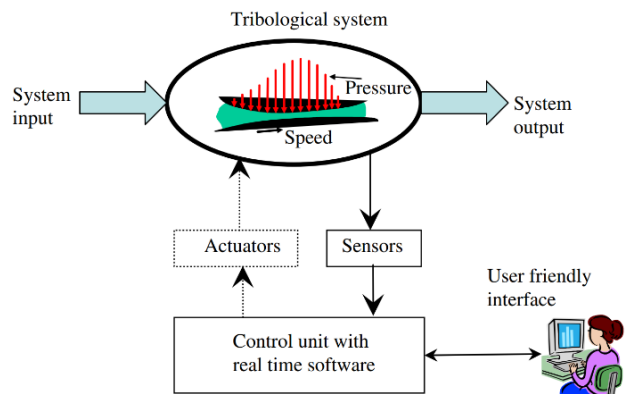
**Fig. 21.** Sketch and block diagram of the active aerostatic thrust bearing with a conicity control design at KU Leuven University (from [38]).

Experimental results demonstrated that this is one of the most promising active compensation solutions since the bearing exhibits relatively high bandwidth (300 Hz) and guarantees nano-positional accuracy.

### 9. NUMERICAL MODELS AND ACTIVE COMPENSATION: TOWARDS TRIBOTRONICS SYSTEMS

Many efforts have been made to obtain gas bearings with higher performance. These performance enhancements have made it

possible to extend the use of gas bearing technology to micro and nano-applications. In the last thirty years, modern technologies such as controllers, sensors and electronic calculators have been the main driving force behind the progress in the field of gas lubrication. The use of contemporary electronic calculators has allowed the development of very accurate numerical models which have been successfully used to simulate the performance of air bearings enabling a safer design. Moreover, some of these models have allowed the reliability of gas bearings to be statistically quantified. Finite element models used in combination with statistical approaches, (e.g., Stochastic Response Surface and Montecarlo Methods) have been used to estimate the influences of some parameters such as supply pressure, orifice type and size on bearing performance and reliability [41–43]. Multiphysics models have been employed to model and actively control the interactions between the gas film and the feeding system of a bearing. These models integrated with compensation methods may be considered as the first examples of tribotronic systems. This term, that coined a few years ago at the Division of Machine Element of Leula University of Technology [44], defines a modern branch in the field of Tribology. Tribotronic applications integrate mechatrical and tribological systems in the attempt to improve their performance. This new field stemmed from the relentless demand for more efficient and lower-weight machineries. The aim of this integration is to enhance the performance of passive tribological systems that otherwise would be limited and very sensitive to operating condition changes.



**Fig. 22.** Scheme of a Tribotronic system (from [44]).

Figure 22 illustrates the main elements composing a tribotronic system. Passive

tribological systems have different functional inputs, e.g., speed, torque, load, and friction and wear as outputs. Tribotronics systems consist of integrating tribological systems with sensors, actuators and real time controllers to optimise their performance and reliability.

## 10. FUTURE PERSPECTIVES

The accuracy of current numerical models along with the integration of active compensation systems represent a solid base to improve further air bearing performance and try to extend their use to many others micro and nano-applications. Indeed, air bearings are an efficient and environmentally friendly solution regarding positioning systems. Air bearings can be adopted in harsh environments providing zero loss, friction and wear. Nowadays, the major obstacle in developing these tribotronic systems is the cost of these complex systems. However, technological progress and its continuous upgrade offer the hope that these costs may be significantly reduced in the future. Despite this, air bearing technology is anyway moving forward. One of the most recent idea is by the New-Way company, which is trying to use gas bearings as alternative turbochargers for auto-vehicles [45]. This kind of application is promising to remarkable future advantages. The use of air in place of oil may minimise costs, emissions and maintenance time and simultaneously increase vehicle performance.

## 11. CONCLUSIONS

Friction and wear are tribological phenomena that have produced significant economic damage and environmental issues throughout our history. Because of their remarkable performance, using gas-lubricated systems represents a worthy and eco-friendly solution to reduce losses, wear and friction within machine elements.

This paper has presented a historical review of gas lubrication from its accidental discovery until today. The review first has considered the milestones of fluid mechanics, i.e., mass conservation and momentum equations, which made possible the birth and development of gas lubrication technology. The paper has concluded

by analysing the newest methodologies that are currently employed for enhancing the performance of gas lubricated systems. Because of their features, these solutions may be considered as potential applications in a new field, which is called Tribotronics. The aim of this branch of Tribology is to integrate mechatronic devices with conventional tribological systems for controlling their behaviour, thus achieving higher performance and accuracy. It is hoped that this new insight, combining knowledge of the fields of tribology and mechatronics, will push the boundaries of current technology and will be a significant step towards the reduction of the costs and the alleviation of the environmental issues associated with tribological phenomena.

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