

Research of Influence of Engagement Pairing of the Corrected Worm Gear with Involute Worm on the Life and Contact Pressure

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A B S T R A C T

The impact of meshing type in involute worm gears as well as the effect of tooth correction and their wear are analyzed by the author's own method for calculating worm gear life. Qualitative and quantitative regularities are established regarding the effect of tooth wear on the variation in the maximum contact pressures and minimum life of the gear.

The minimum life of the gear with triple-pair meshing is higher by 1.46 times when compared to that of the gear with double-pair meshing. Following the tooth correction, the minimum gear life is increased by 1.22 times. The maximum contact pressure under the triple-pair meshing conditions is lower by 1.22 times when compared to the double-pair meshing case, and the correction of the gear teeth leads to a reduction in the maximum contact pressure by 1.28 times. Predictive estimation of the specified contact parameters and tribotechnical characteristics of the worm gear was performed using experimentally determined indicators of material wear resistance.

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1. INTRODUCTION

Worm gears are widely used in gear trains in machine design. The calculation of their load capacity is standardized (ISO / TR 14521: 2010, DIN 3996, BS 721, AGMA 6034, etc.). When designing worm gears, there is no difficulty in assessing load capacity and contact strength of teeth. Involute worm gears ensure complete contact between the worm and the gear teeth, and thus have a higher load carrying capacity. They are used when high hardness and low

surface roughness are required. Due to the fact that the sliding friction in a worm gear causes wear of the gear teeth, it is vital to be able to predict the life of worm gear teeth and the occurring contact pressures already at the stage of gear design. Accordingly, in [1,2] the effect of loading on the contact parameters at EHDL was investigated. However, the literature of the subject provides hardly any applicable calculation methods or standards for assessing the life of worm gears. In fact, only in [3-6] the wear of worm gear teeth with

elastohydrodynamic lubrication is analyzed by the Archard equation for abrasive wear. However, this type of wear cannot be applied to the above-mentioned type of lubrication. The method proposed in [7], which is based on empirical dependencies for predicting the life and wear of worm gears, is also less effective. The formulas presented in [7] have a limited scope of practical application. The behavior of worm gears during loading, including the occurrence of contact pressures, was investigated in [8]. In [9], a model for predicting tooth wear along the contact line is presented. The model takes account of lubricant thickness, which is also based on Archard's law of abrasive wear. The contact pressures and wear are calculated. Recommendations regarding the design of worm gears are given in [10].

It should be emphasized that the aforementioned studies do not describe the effects of the type of meshing and the wear-dependent correction of gear teeth and tooth curvature radius of the working profile on the life and contact strength of worm gears. These factors were only taken into consideration in the generalized method developed by the author [12-14]. In [11-14], worm gears with double-pair meshing of the worm and gear teeth were investigated by this method.

This paper presents the results of a study investigating the effects of meshing conditions and technological (tooth correction) and operating (tooth wear) factors on the life and contact strength of involute worm gears.

2. THE SOLUTION METHOD

The essence of the phenomenological method based on a mathematical model of the sliding friction and fatigue wear of tribological system elements is described in detail in [13-16]. Its components are:

- a) Mechanical model of the destruction process of destruction of surface layers of materials of the sliding friction tribosystem according to the theory of friction fatigue mechanism.
- b) Mathematical model taking into account the basic laws of micro- and macromechanics of tribocontact interaction during sliding friction.

- c) An experimental method of estimating the wear resistance of tribological pairs materials to determine the basis parameter and wear resistance indicators in this mathematical model.
- d) Calculation methods for contact parameters in different types of tribomechanical systems (slide bearings, guides, gears, worm gears, cam and friction mechanisms, brakes, etc.).

Below are presented only the main design formulas.

According to [13,14], the function of linear wear of the gear teeth per one revolution, considering the impact of tooth wear on the changes in initial contact pressures, can be calculated by the following formula:

$$\hat{h}_{2j} = \frac{\nu_j t'_{jh} \left(f p_{jh \max}^{(w)} \right)^{m_2}}{C_2 (\tau_{s2})^{m_2}}, \quad (1)$$

$$p_{jh \max} = 0.564 \sqrt{N'/bw\theta\rho_{jh}}, \\ 2b_{jh} = 2.256 \sqrt{\theta N' \rho_{jh} / bw}, \quad (2)$$

where $\theta = (1 - \mu_1^2)/E_1 + (1 - \mu_2^2)/E_2$.

$$\rho_{jh} = \frac{\rho_{1j} \rho_{2jh}}{\rho_{1j} + \rho_{2jh}}, \quad t'_{jh} = 2b_{jh} / \nu_j.$$

$$\rho_{2jh} = \rho_{2j} + \lambda_h \sum_{jn=1}^B h_{2jn}. \quad (3)$$

The sliding rate is:

$$\nu_j \approx \nu'_j = \frac{\omega_1 x}{\cos \gamma_A}, \quad (4)$$

where $\tan \gamma_A = mz_1 / 2x$, $\omega_1 = \pi n_1 / 30$.

The coordinate x is in the range of $x_A < x < x_B$. Accordingly, $x_A = r_{f_1} + 0.2m$, $x_B = r_{a_1}$ (Fig. 1).

The meshing force N' is determined by the formula:

$$N' = \frac{2T}{d_1 \cos \alpha_{px} \sin(\gamma + \rho')}, \quad (5)$$

where $T = 9550 \cdot 10^3 (N/n_1)$, $\rho' = a \tan(f/\cos \alpha)$, $d_1 = qm$.

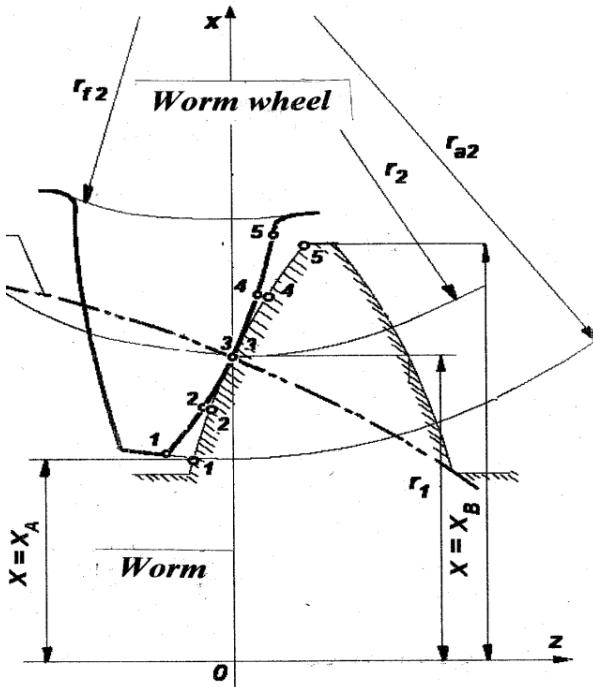


Fig. 1. The location of the contact points on the worm and worm gear profiles.

The curvature radius of an involute worm:

$$\rho_j = \frac{\rho_{1j}\rho_{2j}}{\rho_{1j} + \rho_{2j}}. \quad (6)$$

where ρ_{1j} , ρ_{2j} [14].

According to the proposed block-cumulative method, the accumulated linear wear h_{2jn} of the worm gear teeth is calculated as the product of wear in every block (cycle) of interaction, depending on the number of blocks B . All parameters are maintained constant in every single interaction. Accordingly,

$$h_{2jn} = \sum_1^{n_{2*}} \hat{h}_{2jn}, h_{2jn} = \sum_1^B h_{2jB}, \quad (7)$$

where $h_{2jB} = \sum h'_{2j}$.

Thus, the linear wear h_{2j} at the points of the tooth profile described by the coordinates x during a single interaction, with the contact conditions $(p_{j\max}, 2b_j)$ maintained constant in every block, is calculated using the following formula:

$$\hat{h}_{2j} = \frac{v_j t'_j \left(f p_{j\max}^{(w)} \right)^{m_2}}{C_2 (\tau_{s2})^{m_2}}, \quad (8)$$

where $t'_j = 2b_j^{(w)} / v_j$.

When the maximum accumulated linear wear of the gear teeth amounting to $h_{2jn} = h_{2*}$ is reached, it is possible to calculate a corresponding number of revolutions, n_{2*} . Considering the maximum number of revolutions, gear life is determined in the following way:

$$t_* = n_{2*} / 60n_2. \quad (9)$$

It should be noted that the gear life t_* will be different, depending on the point of the tooth profile. Accordingly, from the calculations we select $t_{*\min}$ to determine the minimum life of the gear. The results demonstrate that the minimum gear life occurs at point $j=1, x=18$ mm (the top tooth entrance into engagement), whereas at point $j=5, x=26$ mm (at the tooth base) the gear life is maximal.

It is widely known that in the case of worm gears, correction can only be applied to the worm gear teeth. The center distance is:

$$a_k = a + x_2 m, \quad (10)$$

where $a = (d_1 + d_2)/2$, $d_2 = z_2 m$, $z_2 = u z_1$, $x_2 = 0 \dots 1.0$.

The reference diameter of the worm in a corrected gear is:

$$d_{w1} = d_1 + 2x_2 m. \quad (11)$$

Other geometrical parameters are determined in accordance with the formulas for uncorrected worm gears.

3. NUMERICAL SOLUTION

The calculations were performed for a typical worm gear described by the following data: $N = 3.5$ [kW], $n_1 = 1410$ [rpm], $m = 6$ [mm], $z_1 = 2$, $u = 25.5$, $f = 0.05$ - boundary friction, $q = 8$, $z_2 = 51$, $d_1 = 48$ [mm], $d_2 = 306$ [mm], $a = 177$ [mm]; worm - hardened steel grade 45 (HRC 50) described by $E_1 = 2.1 \cdot 10^5$ [MPa], $\mu_1 = 0.3$; worm gear ring - bronze CuSn6Zn6Pb6 described by $E_2 = 1.1 \cdot 10^5$ [MPa], $\mu_2 = 0.34$; $C_2 = 7.6 \cdot 10^6$, $m_2 =$

0.88 – the values were determined by the author in previous studies using the method [2,14]; $\tau_{s2} = 75$ [MPa]; for $j = 1; 2; 3; 4$ and 5, respectively $x = 18; 20; 22; 24$ and 26 [mm]; $h_{2*} = 0.5$ [mm]; $\lambda_h = 100$ [14]; $B = 10(60n_1/u) = 33177$ revolutions (10 hours of work) [14]; with double-pair and triple-pair meshing.

Obtained numerical results are given in Figs. 2–6. As a result of the numerical solution, the effect of the meshing engagement between the worm and the worm gear teeth could be analyzed. As a rule, triple-pair meshing is predominantly typical of worm gears, and the case of double-pair meshing is intermediate.

Figure 2 shows the maximum pressures $p_{1\max}, p_{5\max}$ at two points of contact for two conditions: $p_{j\max} = \text{const}$ (the effect of tooth wear is omitted) and $p_{j\max} = \text{var}$ (tooth wear is considered).

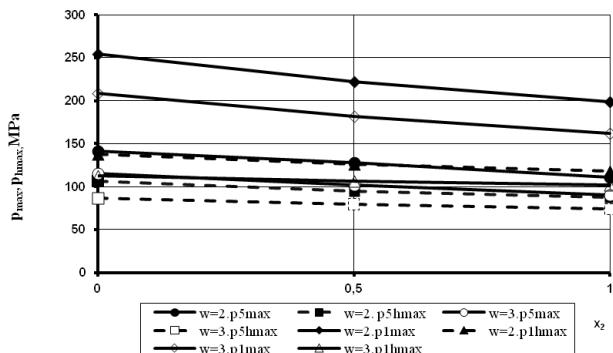


Fig. 2. Variations in the maximum contact pressures.

An analysis of the results shown in the Fig. 2 leads to the following conclusions:

- under the triple-pair meshing conditions is significantly lower than in the double-pair meshing case;
- the application of tooth correction is recommended only for the positive values of x_2 (cf. [14] for the case when $x_2 < 0$);
- tooth wear has a considerable effect on $p_{j\max}$. However, it has practically no effect on the minimum gear life t_{*min} .

Variations in the initial contact pressure p_{\max} during a meshing cycle in the first revolution of the gear, depending on the meshing type w and the tooth correction factor p_{\max} , are shown in Fig. 3.

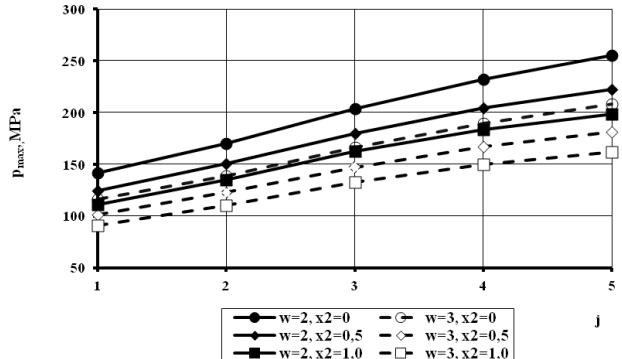


Fig. 3. Variations in p_{\max} during a meshing cycle.

Figure 4 shows the variations in $p_{h\max}$ when the maximum tooth wear h_{2*} amounting to 0.5 mm is reached.

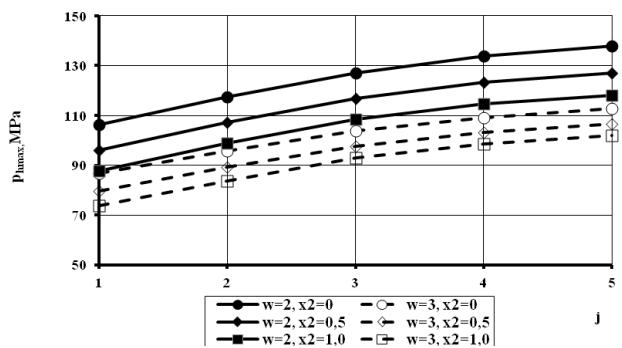


Fig. 4. Variations in $p_{h\max}$ when the maximum tooth wear is reached.

It can be observed that p_{\max} increases at both the entry and the exit of engagement. A similar trend can also be observed for $p_{h\max}$ when the tooth wear h_{2*} is 0.5 mm. An increase in the reduced radius of curvature resulting from the wear of the gear teeth leads to a significant decrease in p_{\max} for both double- and triple-pair meshing conditions.

Figure 5 shows the effects of the applied meshing type ($w = 2; 3$) and tooth correction ($x_2 > 0$) on the minimum gear life t_{*min} .

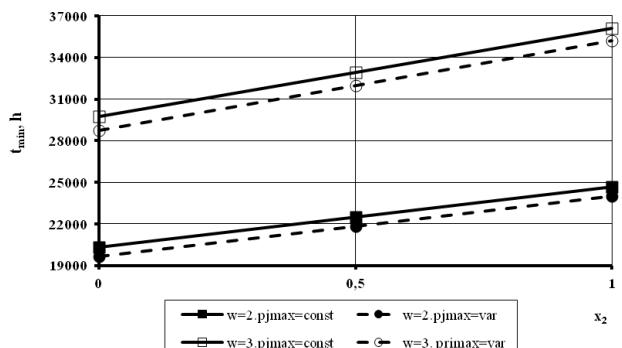


Fig. 5. Gear life.

It should be noted that t_{\min} is higher under the triple-pair meshing, and the tooth correction also leads to an increase in the gear life. However, the change in the contact conditions (particularly $p_{jh\max}$) due to the wear of the gear teeth has practically no effect on the minimum gear life t_{\min} (the gear life does not decrease to any significant extent). This fact can be explained by the presence of two competitive processes when $p_{j\max} = \text{var}$:

- both the friction path of $2b_{jh}$ and the tribocontact time of $t'_{jh} = 2b_{jh}/v_j$ increase, which leads to an increase in the linear tooth wear h_{j2B} during their interaction in block B ;
- the contact pressures $p_{jh\max}$ decrease (Fig. 4) due to an increase in the reduced curvature radius.

An analysis of the results demonstrates that the pressure $p_{jh\max}$ has a more significant effect on the minimum gear life than t'_{jh} . Generally, the minimum gear life t_{\min} practically does not change, compared to the case when $p_{j\max} = \text{const}$.

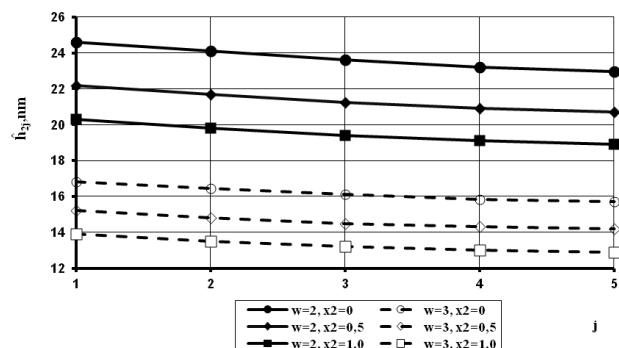


Fig. 6. Linear wear of the gear tooth profile per hour.

Figure 6 shows the variation in the linear tooth wear \hat{h}_2 per hour at individual meshing points when $p_{\max} = \text{const}$. The results also demonstrate that at $p_{\max} = \text{var}$, the wear \hat{h}_2 (in the last interaction block) is slightly higher.

4. CONCLUSIONS

- The purpose of the publication on the study of the effect of parity of the gear in the worm

gear, correction of the teeth and wear on their contact strength and resource has been fully accomplished.

- The type of meshing has a significant effect on $p_{j\max}$, $p_{jh\max}$ and t_{\min} . The maximum contact pressures $p_{j\max}$ decrease by almost 1.22 times, while the maximum tribocontact pressures $p_{jh\max}$ decrease by 1.2 times.
- Following the worm gear tooth correction by $x_2 = 0 \dots 1.0$, $p_{j\max}$ decreases by 1.28 times, whereas $p_{jh\max}$ decreases by 1.1 up to 1.21 times.
- After changing the meshing type, the minimum gear life increases by 1.46 times, whereas after the change in x_2 – it only increases by 1.22 times.
- The results confirm that when calculating the life of worm gears, one must consider the following factors: technological (gear tooth correction), meshing engagement (double- and triple-pair), operational (gear tooth wear), tribological (the coefficient of friction, wear resistance of gear materials).

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NOMENCLATURE

- a [mm] is the center distance in an uncorrected gear,
- b_2 [mm] is the width of the worm gear,
- C_k, m_k [-] are the indicators of wear resistance of the tribological pair materials under selected wear conditions,
- d_1 [mm] is the reference diameter of the worm,
- d_2 [mm] is the reference diameter of the worm gear,
- e_{pA} [mm] is the distance of j -th point from the contact point,
- f [-] is the coefficient of sliding friction,
- q [-] is the diametric quotient of the worm gear,
- $\hat{h}_{2,jn}$ [mm] is the linear wear of the gear teeth during a single interaction, reduced due to changes in $\rho_{2,jh}, t'_{jh}, P_{jh\max}$,
- $h_{2,jB}$ [mm] is the wear of the gear teeth during a single cycle of interaction,
- h_{2*} [mm] is the maximum wear of the worm gear teeth,
- j [-] is the point of contact between the kinematic pair elements (worm – worm gear),
- m [mm] is the axial modulus of meshing,
- n_1 [rpm] is the number of revolutions of the worm,
- n_2 [rpm] is the number of revolutions of the worm gear per minute,
- n_{2*} [rot] is the number of revolutions of the worm gear when the maximum worm gear teeth wear h_{2*} is reached,
- N' [N] is the meshing force,
- N [kW] is the transmitted power,

$p_j = p_{j\max}$ [MPa] are the maximum contact pressures determined by the Hertz formula, depending on the number of meshing pairs w of the worm gear teeth,

$p_{jh\max}$ [MPa] are the maximum current tribocontact pressures,

r_{f1} [mm] is the radius of a circle of the worm cavity,

r_{a1} [mm] is the radius of a circle of the worm thread prongs,

t'_{jh} [sec] is the time of contact between the meshing elements at j -th point on the friction path with a length equal to the contact area width $2b_{jh}^{(w)}$,

t'_j [sec] is the time of contact between the meshing elements at j -th point on the friction path with a length equal to the contact area width $2b_j^{(w)}$,

T [Nmm] is the torque transmitted by the worm,

u [-] is the gear ratio,

v [mm/sec] is the sliding rate at j -th point of contact between the kinematic pair elements,

v'_j [mm/sec] is the sliding rate during worm gear revolution,

w [-] is the number of meshing pairs between the worm thread and the worm gear teeth,

x_2 [-] is the correction coefficient,

z_1 [-] is the number of worm threads,

z_2 [-] is the number of teeth on the worm gear,

α [degree] is the pressure angle,

α_{pxj} [degree] [14],

γ [degree] is the lead angle,

λ_h [-] is the non-dimensional coefficient of wear,

μ_k [-], E_k [MPa] are Poisson's ratio and Young's modulus of the worm gear material, respectively,

ρ_j [mm] is the reduced curvature radius between the worm coil and the gear tooth at j -th point of meshing,

ρ [mm] is the reduced radius of curvature of the involute worm gear,

ρ_{jh} [mm] is the reduced radius of curvature of the involute worm gear due to tooth wear; the wear of the steel worm is omitted,

ρ_{2jh} [mm] is the radius of curvature of the Archimedes worm gear,

ρ' [degree] is the friction angle,

τ_{s2} [MPa] is the temporary shear strength of the worm gear material,

θ [1/MPa] is the Kirchhoff modulus,

ω_1 [1/sec] is the angular velocity of the worm.