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RESEARCH

The Mathematical Model of Motion Trajectory of Wear Particle Between Textured Surfaces

V. Marchuk[a], M. Kindrachuk[a], Ya. Krysak[a], O. Tisov[a,*], O. Dukhota[a], Y. Gradiskiy[b]

[a]*National Aviation University, Kyiv, Ukraine*
[b]*Petro Vasylenko National Technical University of Agriculture, Kharkiv, Ukraine*

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* Corresponding author:

Oleksandr Tisov
E-mail: tissov@ukr.net

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A B S T R A C T

An effort was made to obtain a simplified mathematical model of wear particle motion in a magnetic field formed between flat and dimpled surfaces during the friction process. The magnetic field's experimental values on the dimple edge and inside the dimple for several materials are presented. As a direct result of the internal magnetic field's excitation at the edges of indentations, debris falls into the dimple. This reduces the wear rate by decreasing the number of abrasives in the friction interface. Apparent results expand ideas about the phenomena that naturally occur during the friction of textured surfaces.

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1. INTRODUCTION AND STATE OF THE ART

One of the promising solutions to improve the durability of machine elements is surface texturing. The essence of this method is the formation of specially patterned indentations or dimples. There are various technologies of surface texturing, particularly laser texturing [1-3], vibro-shock [4, 5], vibration texturing [6], reactive ion-beam etching [7], and other methods.

The wear mechanism of textured surfaces depends on many factors. Dimples are constant sources for lubricant supply. They help to recover lubricating film [8] and are the place where wear debris fall and accumulate. The efficient surface texture allows designing a tribotechnical surface with high performance. The formation of dimples also reduces the run-in period [9-11].

Along with the formation and destruction of friction-induced structures, the excitation of an internal magnetic field at dimple edges is also essential. These magnetic fields affect the motion of wear particles. Experimental studies show that dimple edges form their own magnetic field. Depending on coupled materials, the magnetic field at the edges is 10–26% higher than magnetic field between the dimples. The magnetic induction on edge may be as high as 44–62 mT. At

the bottom of a dimple, it abruptly decreases down to 13–17 mT [8]. We proved this by measuring the magnetic field induction for a set of materials (Fig. 1). The magnetic field change over the dimpled surface is in Fig. 2.

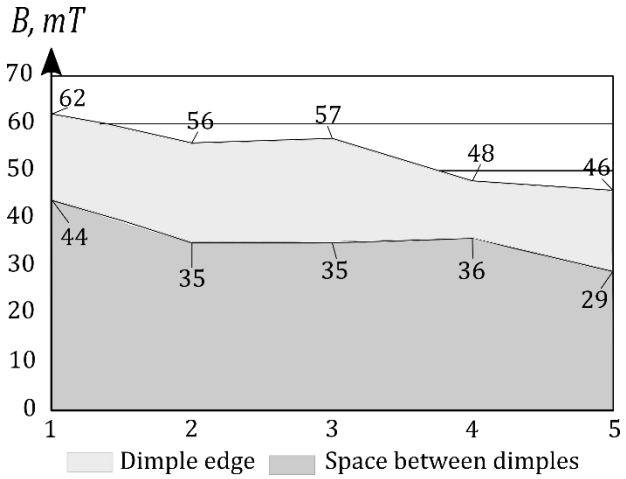

Fig. 1. Measured values of magnetic induction B for various dimpled materials: 1 – cast iron vs cast iron; 2 – steel 0.14C17Cr2Ni- steel 0.14C17Cr2Ni; 3 – steel 0.3CCrMnSi vs steel 0.3CCrMnSi; 4 – steel 0.3CCrMnSi-steel 1045; 5 - steel 0.3CCrMnSi-steel 1030

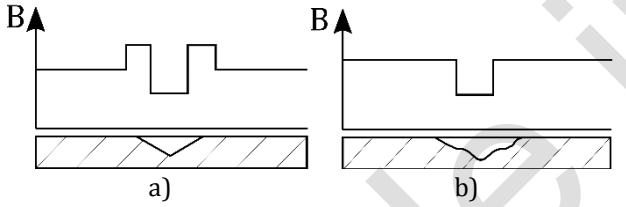

Fig. 2. Distribution of magnetic field over the dimpled surface: a – newly produced, b – worn off surface.

Here was decided to study the effect of friction-induced magnetic field on the removal wear debris and how this finally will influence wear resistance of textured surfaces. According to [12], ferromagnetic particles will be attracted to dimples' edges and fall into them. This condition is described by (1):

$$\frac{\mathrm{m}\cdot\theta\cdot B_e^2}{3\pi\cdot\mu_0\cdot\mu\cdot\rho_\mathrm{m}\cdot\eta_\mathrm{l}\cdot r\cdot V}e^{-4\theta S}>1. \qquad (1)$$

m – the mass of the particle;
r – estimated radius of the particle;
V – particle velocity;
η_l – dynamic viscosity of the liquid;
B_e – magnetic flux density on the edges of dimples;
θ – dimple aspect ratio (width/depth);
S – the distance between contiguous surfaces;
μ_0 – $4\pi\times10^{-7}$ H/m– magnetic constant;
μ – relative magnetic permeability of lubricant molecules, showing how much the magnetic permeability of lubricant differs from the magnetic permeability on the dimple edge; ρ_m – wear particle density.

The dependence of the edge B_e on θ (aspect ratio of dimple: the ratio of width to depth) is established. As the θ increases, the magnetic flux density of the edges of the holes decreases exponentially. The consequence of this is to increase the angle of dimple side surface inclination up to 180° [12].

Many studies have focused on the effects of external electromagnetic radiation on the tribosystem. The obtained model of ferromagnetic microparticle motion in a high-frequency magnetic field [13] allowed estimating external factors' influence on the conditions of microparticle capturing by the magnetic trap:

$$m\ddot{\vec{r}}+\sigma(\ddot{\vec{r}}-\vec{V}_0)= -\frac{8}{9}\frac{mM^2a^2}{m^2\omega^2+\sigma^2}(x\vec{e}_x+y\vec{e}_y+9z\vec{e}_z)\cos(\omega t). \qquad (2)$$

m – the mass of the particle;
σ – dissipation factor;
$\ddot{\vec{r}}$ – second time derivative of space coordinate;
$\vec{V}_0$ – liquid flow velocity;
$\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$ – unit vectors along axis *x, y, z*;
a – a factor, proportional to the electric current in electric magnet coils;
M – magnetic moment (J/T);
ω – frequency of magnetic field;

The motion of a microparticle is characterized by maximum oscillation amplitude of the transition process time, the amplitude of oscillations in magnetic trap and coordinates of the point around which the particle oscillates.

A mathematical model of the external magnetic field influence on particle motion was considered from reference [14], which describes the process of particle motion in a disk-type separator in the projection along X and Y axes (3).

$$\frac{d^2x}{dt^2}=-\frac{9\eta}{2r^2\rho}\frac{dx}{dt}+\mu_0\chi\alpha H_0^2\exp(-2\alpha y)\cos(\beta)$$
$$\frac{d^2y}{dt^2}=-\frac{9\eta}{2r^2\rho}\frac{dy}{dt}+\mu_0\chi\alpha H_0^2\exp(-2\alpha y)\sin(\beta)+g \qquad (3)$$

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η – dynamic viscosity of air;
χ – specific magnetic susceptibility of particle;
H_0 – magnetic field intensity on the surface of material;
α – coefficient of magnetic field inhomogeneity;
y - distance to the particle;
g – gravity.

The differential equations system's solution under different initial conditions makes it possible to study particle trajectory in a magnetic disk-type separator and the influence of parameters on the quality of separation.

The internal electromagnetic field that develops in the friction zone may reduce wear of machine elements. Knowing how this affects wear process of textured surfaces will disclose one more factor to be controlled to improve friction couples' wear resistance. The study of the dimple's internal magnetic field influence on wear debris collection could present particular scientific interest.

2. OBJECTIVES

This study aims to develop a mathematical model of the trajectory of wear particles in the lubricated tribological contact of the textured (dimpled) surface with flat surface taking into account the hydraulic forces and magnetic field generated in the friction interface.

3. RESULTS AND DISCUSSION

To study the mechanism of wear products accumulation in the dimple and to better understand what is going there, let's consider the problem of ferromagnetic particle (wear particle) motion between dimple and flat surface that are in reciprocating friction mode (Fig. 3)

Ferromagnetic wear particle engaged by the magnetic field between two contacting surfaces, is also magnetized. The other magnetic field arises at the edge of the dimple with intensity H_e. Dimple edge magnetic field is much stronger than both particle and roughness asperities magnetic fields. (see Fig. 3).

Interaction forces between dimple and wear particle magnetic fields form the resultant inhomogeneous magnetic field, causing the ponderomotive force F_p which pulls the particle towards dimple center.

$$F_{\mathrm{p}} = \mu_0 \cdot \chi \frac{m}{\rho} H \cdot \mathrm{grad}H \,. \tag{4}$$

$$\mathrm{grad}H = \frac{\partial H}{\partial x}\vec{i} + \frac{\partial H}{\partial y}\vec{j} \,. \tag{5}$$

m – mass of wear particle, kg;
H – strength of magnetic field in particle area;
ρ – density of lubricating oil, kg/m³.

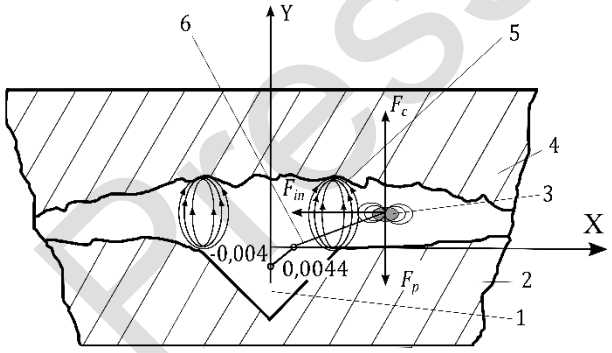

Fig. 3. Model of wear particle trajectory of motion into the: 1– dimple; 2– counterpart; 3– wear particle with the intrinsic magnetic field; 4– specimen; 5– magnetic flux lines nearby the dimple edge; 6- the trajectory of wear particle motion.

Besides, some other forces act on a ferromagnetic particle with mass m, namely:

1. Stokes drag (F_d)

$$F_{\mathrm{d}} = 6\pi \cdot \eta_{\mathrm{l}} \cdot r \cdot \nu = \frac{9\eta_{\mathrm{l}} \cdot m}{2r \cdot \rho} \frac{dy}{dt} \,. \tag{6}$$

ν – relative speed of particle relative to axes Y, (ν=dy/dt, m/sec);

2. Inertia force (F_{in})

$$F_{\mathrm{in}} = m \cdot a_{\mathrm{i}} \,. \tag{7}$$

a_i –particle deceleration, which under the action of magnetic flux is assumed to be uniform.

Using well-known physical relations [15] this value may be determined as:

$$a_i = \frac{v_i^2}{2L} \,. \tag{8}$$

L – deceleration path, v_i – inertial velocity of wear particle caused by reciprocating motion of friction surfaces.

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So:

$$F_{\text{in}} = m\frac{v_{\text{i}}^2}{2L} = \frac{m}{2L}\left(\frac{dx}{dt}\right)^2. \quad (9)$$

Studies [16] of the forces acting on the particle, led to the conclusion that magnetic deposition occurs due to competition between magnetic force and Stokes drag. For particles less than 0.01-10 μm in size, gravity force may be neglected because it is up to 9 orders of magnitude smaller than the Stokes drag.

According to [15], resolving the equation of particle motion on coordinate axes, we obtain differential equations of particle motion:

$$m\frac{dv}{dt} = \sum_{i=1}^{n} F_i \rightarrow \begin{cases} m\dfrac{d^2y}{dt^2} = F_{\text{d}} - F_{\text{n}} \\ m\dfrac{d^2x}{dt^2} = -F_{\text{in}} \end{cases}. \quad (10)$$

After substituting into the system (11) of the corresponding values of forces projections on axis OX and OY and simplifications, we obtain

$$\begin{cases} \dfrac{d^2y}{dt^2} = \dfrac{9\eta_{\text{l}}}{2r^2\rho}\dfrac{dy}{dt} - \mu_0 \cdot \chi \dfrac{1}{\rho} H \dfrac{\partial H}{\partial y} \\ \dfrac{d^2x}{dt^2} = -\dfrac{1}{2L}\left(\dfrac{dx}{dt}\right)^2 \end{cases}. \quad (11)$$

Before solving the equation, note that the change in magnetic field intensity between the contacting surfaces is known:

$$H = H_e e^{-2\theta y}. \quad (12)$$

H_e – the intensity of the magnetic field at the edge of the dimple;

y – distance to the particle;
θ – aspect ratio of the dimple.

Taking this into account, we may stand on the following:

$$H \cdot \frac{\partial H}{\partial y} = H_e e^{-2\theta \cdot y} \cdot \frac{\partial}{\partial y} H_e e^{-2\theta \cdot y} =$$

$$= -2\theta \cdot H_e^2 e^{-4\theta \cdot y} \cong -2\theta \cdot H_e^2 (1 - 4\theta \cdot y). \quad (13)$$

When y values are low:

$$e^{-4\theta \cdot y} \cong 1 - 4\theta \cdot y.$$

Finaly, let's write the system (11) as follows:

$$\begin{cases} \ddot{y} - \dfrac{9\eta_{\text{l}}}{2r^2\rho}\dot{y} + \dfrac{8\mu \cdot \chi \cdot H_{\text{e}}^2 \cdot \theta^2}{\rho} y = \dfrac{2\mu_0 \cdot \chi \cdot H_{\text{e}}^2 \cdot \theta}{\rho} \\ \ddot{x} + \dfrac{1}{2L}\dot{x}^2 = 0 \end{cases}. \quad (14)$$

The first differential equation of the system (15) is an inhomogeneous linear equation of second order. Let's solve the homogeneous equation. For this purpose, we use the characteristic equation (15): $\lim\limits_{x \to \infty} f(x)$

$$k^2 - \frac{9\eta_{\text{l}}}{2r^2\rho}k + \frac{8\mu_0 \cdot \chi \cdot H_{\text{e}}^2 \cdot \theta^2}{\rho} = 0. \quad (15)$$

Substituting the values of physical quantities into a characteristic equation, we obtain its solution

$$k_1 = 0{,}75, \quad k_2 = 0{,}0003$$

So, the general solution of a homogeneous differential equation is:

$$y_1 = C_1 \cdot e^{0{,}75 \cdot t} + C_2 \cdot e^{0{,}0003 \cdot t}. \quad (16)$$

Partial solution of the inhomogeneous equation is:

$$y_2 = \frac{1}{4\theta} = \frac{1}{2}, \quad \text{at} \quad \theta = \frac{1}{2}.. \quad (17)$$

Thus, the general solution of the inhomogeneous differential equation of system (3) is:

$$y = C_1 \cdot e^{0{,}75 \cdot t} + C_2 \cdot e^{0{,}0003 \cdot t} + \frac{1}{2}. \quad (18)$$

Taking into account the fact that for the little values of t, $e^{\alpha \cdot t} \cong 1 + \alpha \cdot t$, we will get:

$$y = C_1 + C_2 + (0{,}75C_1 + 0{,}0003C_2)t + \frac{1}{2}. \quad (12)$$

Let's choose initial conditions to determine values of constants. If $t = 0$ the particle is engaged by the edge magnetic field. This is a point with coordinates $x = 0{,}005$, $y = 0{,}0005$ with velocities $\dot{x} = 0{,}01$ and $\dot{y} = 0$. Substituting the boundary conditions into the solution, we obtain a system of equations for determining the constants C_1 and C_2

$$\begin{cases} 0{,}0005 = C_1 + C_2 + 0{,}5 \\ 0 = 0{,}75C_1 + 0{,}0003C_2 \end{cases}. \quad (20)$$

$$C_1 = 0{,}0022; \; C_2 = 0{,}5027.$$

Thus, the general solution of the inhomogeneous differential equation of system (3) is the next:

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$$y = 0,018t + 0,0005 . \quad (21)$$

A differential equation $\ddot{x} + \frac{1}{2L}\dot{x}^2 = 0$, after substitutions $z = \dot{x}$, $\dot{z} = \ddot{x}$ reduces to an equation with separated variables:

$$\frac{dz}{dt} = -\frac{1}{2L}z^2, \ \frac{1}{z} = \frac{t}{2L} + C_1,$$

$$\frac{dx}{dt} = \frac{2L}{t + 2C_1L}$$

Finally, we obtain:

$$x = 2L \cdot \ln\left|t + 2C_1L\right| + C_2 . \quad (22)$$

If $t=0$; x=0,005; $\dot{x} = 0,01$; $0,01 = \frac{1}{2C_1}$; C_1=50.

If C_1=50, $x = 2L \cdot \ln\left|t + 100 \cdot L\right| + C_2$.

Let $L = \frac{1}{100}i$,

then $x = 0,02\ln\left|t+1\right| + C_2 = 0,02t + C_2$,

as far as for small t, $\ln\left|t+1\right| \cong t$.

So, C_2=0,005.

The final solution of the system of differential equations is:

$$\begin{cases} x = 0,02t + 0,005 \\ y = 0,018t + 0,0005 \end{cases}. \quad (23)$$

From the first parametric equation, we determine t and substitute it to the second equation. We obtain the equation of wear particle motion:

$$y = 0,9x - 0,004 . \quad (24)$$

In the rectangular Cartesian coordinate system, this equation of motion is satisfied by the points of line 6 (see Fig. 1).

4. CONCLUSION

Due to the excitation of internal magnetic fields at the edges of dimples, wear particles fall to the bottom, where they collect, preventing severe abrasive wear around the indentations. Predicting and controlling this process may help to select the dimples shape and texture pattern, promote the effective removal of wear debris from friction interface. This simplified mathematical model may assist in finding a preliminary solution to the problem.

REFERENCES

[1] I. Etsion, *State of the Art in Lazer Surface Texturing,* Journal of Tribology, vol. 127, pp. 248–253, 2005, doi: 10.1115/1.1828070

[2] A. Hoppermann, M. Kordt, *Tribological Optimisation Using Laser-Structured Contact Surfaces,* Oelhydraulik und Pneumatik, vol. 46, pp. 560-564, 2002.

[3] G. Dumitru, V. Romano, H.P. Weber, S. Pimenov, T. Kononenko, J. Hermann, S. Bruneau, Y. Gerbig, M. Shupegin, *Laser Treatment of Tribological DLC Films,* Diamond and Related Materials, vol. 12, iss 3-7, pp. 1034–1040, 2003, doi: 10.1016/S0925-9635(02)00372-2

[4] A. Greco, S. Raphaelson, K. Ehmann, Q. J. Wang, C. Lin, *Surface texturing of tribological interfaces using the vibromechanical texturing method,* Journal of Manufacturing science and engineering, vol. 131, iss, 6, pp. 1-8, 2009, doi: 10.1115/1.4000418

[5] V. Marchuk, M. Kindrachuk, A. Kryzhanovskyi, *System analysis of the properties of discrete and oriented structure surfaces*, Aviation, vol. 18, iss. 4, pp. 161–165, 2014, doi: 10.3846/16487788.2014.985474

[6] Ju.G. Shnejder, *Maintenance properties of elements with regular microrelief*, Leningrad: Machine engineering, 1982. (in Russian)

[7] X. Wang, K. Kato, *Improving The Anti-Seizure Ability of SiC Seal in Water with RIE Texturing*, Tribology letters, vol. 14, pp. 275–280, 2003, doi: 10.1023/A:1022650813314

[8] V.Ye. Marchuk, M.V. Kindrachuk, V.I. Mirnenko, R.G. Mnatsakanov, A.O. Kornienko, O.V. Bashta, S.V. Fedorchuk, *Physical interpretations of internal magnetic field influence on processes in tribocontact of textured dimple surfaces*, Journal of Nano- and Electronic Physics, vol. 11, iss. 5, pp. 1–5, 2019, doi: 10.21272/jnep.11(5).05013

[9] D.A. Volchenko, M.V. Kindrachuk, N.A. Volchenko, S.V. Nikipchuk, M.M. Nasirova, N.B. Klochko, A.O. Yurchuk, *Electrothermomechanical friction of wet elements of automotive disc-shoe brakes.* Journal of Nano- and Electronic Physics, Vol. 12, iss. 6, pp. 1–8, 2020, doi: 10.21272/ jnep.12(6).06020

[10] M.V. Kindrachuk, O.I. Volchenko, D.O. Volchenko, N.O. Volchenko, P.O. Polyakov, A.O. Kornienko, A.O. Yurchuk, *Polymeric materials modified by semiconductor substances in friction units of braking devices*, Journal of Nano- and Electronic Physics, vol. 11, iss. 3, pp. 1–8, 2019, doi: 10.21272/jnep.11(3).03014

[11] V. Marchuk, M. Kindrachuk, O. Tisov, A. Kornienko,

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O. Radko, V. Kharchenko, *Stress-strained state of textured surfaces with selectively indented regions*, Functional Materials, vol. 26. iss. 4, pp. 773-778, 2019, https://doi.org/10.15407/fm26.04.773

[12] V.Ye. Marchuk, Ya.V. Krysak, V.I. Morozov, *Mathematical model of wear products removal in tribocouples with textured dimpled surfaces*, Problems of friction and wear, vol. 82, no. 1, pp. 4–9, doi: 10.18372/0370-2197.1(82).13480 (in Ukrainian)

[13] O.Yu. Horobec, M.M. Potomkin, *Modeling of ferromagnetic particle motion of liquid flow under the action of high-frequency oscillating magnetic field*, Gerald og National university "Lvivska politekhnika", Elektroniks, iss. 681, pp. 178-183, 2010. (in Ukrainian)

[14] M. V. Yatskov, O. I. Mysina, *Mathematical model of the process of magnetic cleaning of substances of impurities*, in Komputer modelling in chemical technologies: first scientific-practical conference with international participation, 12-16 may, 2008, Ukraine, Cherkasy, Cherkasy state technical university, pp. 71– 74. (in Ukrainian)

[15] M.A. Pavlovskyi, *Theoretical mechanics*, Kyiv: Tekhnika, 2002. (in Ukrainian)

[16] A.V. Sandulyak, B.L. Dakhnenko, *Peculatities of magnetic deposition by filtering*, Magnetic hydrodynamics, iss. 2, pp, 123-127, 1989. (in Russian)

Nomenclature

Name	Designation	Units
Particle mass	m	kg
Particle density	ρ_m	kg/m^3
Particle radius	r	m
Particle velocity	V	m/s
Particle relative velocity	v	m/s
Inertial velocity of particle	v_i	m/s
Particle deceleration	a_i	m/s^2
Deceleration path	L	m
Distance to the particle	y	m
Distance between friction surfaces	S	m
Magnetic constant	μ_0	H/m
Relative magnetic permeability of lubricant molecules	μ	-
Magnetic susceptibility	χ	-
Magnetic moment	M	A/m^2
Induction of magnetic field on the edge of dimple	B_e	T
Intensity of magnetic field	H	A/m
Intensity of magnetic field on the surface of discs	H_0	A/m
Intensity of magnetic field on the dimple edge	H_e	A/m
Magnetic field oscillation frequency	ω	Hz
Lubricant density	ρ	kg/m^3
Velocity of liquid flow	V_0	m/s
Dynamic viscosity of liquid	η_l	-
Dynamic viscosity of air	η	-
Dimple aspect ratio	θ	-
Dissipative factor	σ	-
Second time derivative to position	$\ddot{\vec{r}}$	-
Unit vectors along axes x, y and z	$\vec{e}_x, \vec{e}_y, \vec{e}_z$	-
Coefficient, that is proportional to the current in solenoid	a	-
Magnetic field inhomogeneity factor	α	
Gravitation acceleration	g	m/s^2

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