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RESEARCH

Comparative Analysis of Multiple Linear Regression and Artificial Neural Network for Predicting Friction and Wear of Automotive Brake Pads Produced from Palm Kernel Shell

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A B S T R A C T

In this study, comparative analysis of multiple linear regression (MLR) and artificial neural network (ANN) for prediction of wear rate and coefficient of friction brake pad produced from palm kernel shell was carried out. The inputs parameters used for the two models generated using inertia dynamometer were the percentages of palm kernel shell, aluminium oxide, graphite, calcium carbonate, epoxy resin, interface temperature of the brake pad, and work done by brake application. Two model equations were developed using MLR model for predicting wear rate and coefficient of friction while the neural network architecture BR 7 [5-3] 2 was used to predict wear rate and coefficient of friction. The predicted wear rate and coefficient of friction by MLR model were compared with ANN model along with the measured values using statistical tools such as means square absolute error (MAE), root means square error (RMSE), and Nash-Scutcliffe efficiency (NSE). The results revealed that the MLR model outsmarts the ANN model with the values of MAE and RMSE reasonably low and NSE reasonably higher. The best MAE and RMSE values of 0.000 were observed at the three values of measured wear rates and coefficient of friction that matched with the predicted values using MLR compared to -0.0300 and 0.0740 for ANN model. However, the ANN model was equally found suitable for the prediction of wear rate and coefficient of friction of brake pads developed. The implication of these results is that the two models have the capabilities of being used simultaneously when estimating the wear and coefficient of friction of brake pads.

1. INTRODUCTION

Friction and wear performance constitutes two kinds of responses from one tribo-system. The phenomenon of material transfer during sliding is important from both the scientific and practical considerations. As the development of disc brake system is always a big challenge for car manufacturers and suppliers due to the complex nature of wear mechanisms involved in the system [1]. Thus investigating and coming up with model equations for the evolution of the tribo-system (disc, brake pads) we go a long way in solving the friction and wear behaviour problems of automotive brakes. According to Zafaruddin and Dolas [2] disc brake system has a higher wear resistance and easier maintenance as compared to other brake system but due to long repetitive braking leads to brake failure and cause severe wear of brake pad. The authors maintained that due to heavy braking there is formation of hot spots on brake disc and formation of grooves on brake pads and excessive wear leading to failure of brake system. Many efforts have been made in the development of friction/wear models for prediction of wear properties of different engineering materials. Ikpambese et al. [1] stated that Archad was one of the early researchers to develop a linear wear model for metals. Other wear models the authors mentioned were nonlinear wear model for friction materials in a disc brake system developed by Rhee in 1974, and finite element model of a brake pad with particular emphasis to uncertainties.

Multiple linear regression analysis is a technique that allows additional factors to enter the analysis separately so that the effect of each can be estimated. It is valuable for quantifying the impact of various simultaneous influences upon a single dependent variable [3] and [4]. According to Mata [5] multiple linear regression is a method used to model the linear relationship between a dependent variable and one or more independent variables. That the dependent variable is sometimes called the response, and the independent variables the predictors. Artificial Neural Networks (ANNs) are revolutionary computing paradigms that try to mimic the biological brain. These ANNs are modeling techniques that are especially useful to address problems where solutions are not clearly formulated or where the relationships between inputs and outputs are not sufficiently known [6,1].

The performance evaluation of multiple regression and artificial neural network in predicting parameters in engineering have been carried out by many researchers. Abdulkareem and Mohammed [6] investigated the use of artificial neural network for the estimation of wear and temperature in disc and pad. Two types of disc made from aluminum and steel were made to slide against the pad under dry conditions at different time, rotational speed, initial temperature of the disc and load in order to examine the wear. The results showed that the wear and temperature increase with increasing the sliding speed, and load or contact time. The authors concluded that the ANN model was successful in showing its high capability in predicting wear and temperature with the results of the model corresponding with the experimental results. The modelling of wear of organic brake pad using multiple linear regression was carried out by [2]. The inputs into the model equation for predicting wear rate were load, sliding distance and sliding velocity and the authors concluded that it will be helpful for engineers in making changes in the composition which could minimized the wear rate by considering the input parameters used.

The machining forces-tool wear relationship of an aluminium metal matrix composite using multiple regression analysis (MRA) and artificial neural network was investigated by [7]. The results show that force-wear equation derived from MRA was fairly accurate way of predicting the attainment of prescribed tool wear. The effects of cutting parameters on surface roughness and tool wear were investigated in turning novel aluminum alloy ash composite by [8]. The authors concluded that the relationship between cutting responses and input parameters held good for more than 97 % and the model was adequate. Kialashaki and Reisel [9] also used multiple regression and artificial neural network for the development of energy-demand models which were able to predict the future energy demand in the residential sector of the United States. Mata [5] employed artificial neural network and multiple linear regression models for the interpretation of concrete dam behaviour under environmental load. The author concluded that the neural network models were more flexible and proved to be adequate for months with extreme temperatures than the multiple linear regression models with the same variables.

Obviously based on the aforementioned studies multiple regression analysis and artificial neural networks have potential for developing models capable of predicting wear and friction of automotive brake pads produced from palm kernel shell. The previous studies that employed the use of linear regression analysis in the modelling of wear failed to provide the model with the complex synergy of different influence of materials formulation, and operating conditions on wear and friction of the friction materials investigated. In addition little or no information is available from literature on the use of modern and traditional approaches for modelling of friction and wear of automotive brake pads produced from agro wastes. Hence, this study investigated the use of multiple regression analysis in comparison with artificial neural network in predicting wear and friction of automotive brake pads developed from palm kernel fibres.

2. MATERIALS AND METHODS

2.1 Brake pad development

The palm kernel nuts after drying were cracked manually using stones to release the shells and the palm kernel shells (PKS) were sun dried for two weeks to reduce the moisture content to 20 percent. The dried PKS was then ground to powder and sieved to 100 µm particle size using BS 410 standard sieves to be used as fibre for the brake lining formulation. The sieved PKS was added in various percentages with other additives (Table 1) such as aluminium oxide, graphite, calcium carbonate and epoxy resin based on commercial brake pad weight of 176 g. Each of the mixtures was one after the other transferred into a designed mould and pressed at room temperature using 100 KN force for 2 minutes to fabricate full-scale brake samples.

Table 1. Composition of Additives used for Brake Pads Samples from PKS.

Brake Additives	Samples in percentage by weight (% wt)					
	S_1	S_2	S_3	S_4	S_5	S_6
Epoxy- resin	19	40	23	25	15	30
Palm kernel fibres (PKS)	6	10	27	30	35	40
Aluminium oxide	0	6	10	5	5	5
Graphite	5	29	10	5	5	5
Calcium carbonate	70	15	30	35	40	20

The produced brake pads were then cured at 250 °C in digital furnace for 90 minutes after removal from the hydraulic press. The produced samples were finished by polishing them using polisher-grinder with various grinding paper of various sizes to obtain the final brake pad sample as shown in Fig. 1 [10].

Fig. 1. One of the produced samples.

2.2 Inertia dynamometer testing of the produced brake pads

The data needed for the modelling of the produced brake pad samples were tested using Inertia dynamometer located at Anambra State Motor Manufacturing Company (ANAMMCO) Nigeria. The equipment was utilized to generate wear and friction of the produced samples and other operating conditions to be used in the multiple linear regression and artificial neural network models. The produced brake pads were one after the other mounted on the brake assembly unit of the inertia dynamometer.

The initial rotating speed of the driving motor was adjusted to 5.56 m/s with the aid of a variable speed drive called tachometer. The values of wear rate and coefficient of friction were measured (with the aid of the attached sensors) after attaining the set speed via a computer attached to the equipment. The same procedure was repeated for each of the produced samples for speeds of 8.33, 11.11, 16.67, 22.22, and 27.78 m/s to obtain the respective values of wear rate and coefficient of friction. Other parameters of interest obtained by the equipment were brake pads interface temperature, power and stopping time for each of the samples investigated.

2.3 Determination of the predictors for multiple linear regression and artificial neural network models

The independent variables otherwise known as the predictors to be used for the formulations of the two models were thoroughly screened using statistic tool. The three groups of independent variables screened were: materials formulation represented by the percentages of the raw materials shown in Table 1; manufacturing conditions represented by moulding temperature, moulding load, heat treatment time, and moulding time. While the operating conditions were represented by brake interface temperature and work done by brake application. A stepwise forward selection procedure by [11] was adopted for the selection of the best predictors that have a better relationship with wear rate and coefficient of friction which were to be the outputs to the two models. All the inputs variable to the network were one after other correlated with the outputs variables and the straight line regression subsequently fitted. The variables with the most highly correlated variable with R^2 as given in equation (1) was selected.

The straight line regression equations relating wear rate and coefficient of friction with the first selected variable (which is the percentages of the PKS) with the highest R^2 were respectively:

$$W_{rate} = \beta_0 + X_1\beta_1 + E,$$

$$CoF = \sigma_1 + X_1\sigma_1 + E \qquad (1)$$

The hypothesis that $\beta_1 = 0$ was tested by determining if the F statistics for the regression equation was significant by comparing it with $F_{k,n-k-1,\,1-\alpha}$. This procedure was repeated for other predictors to eliminate those independent variables that have no relationship with the outputs.

2.4 Multiple linear regression

Two model equations were formulated for predicting wear rate and coefficient of friction as:

$$W_{rate} = \beta_0 + X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + X_4\beta_4 + X_5\beta_5 + X_6\beta_6 + X_7\beta_7 + E, \qquad (2a)$$

$$CoF = \sigma_0 + X_1\sigma_1 + X_2\sigma_2 + X_3\sigma_3 + X_4\sigma_4 + X_5\sigma_5 + X_6\sigma_6 + X_7\sigma_7 + E \qquad (2b)$$

where, X_1 is the % of palm kernel shell, X_2 is the % of aluminium oxide, X_3 is the % of graphite, X_4 is the % of calcium carbonate, X_5 is the % of epoxy resin, X_6 is the interface temperature of the brake pad, and X_7 is the workdone by brake application as shown in Table 2. While E is the error between the experimental and predicted values. The work done by brake application (X_7) was calculated by multiplying braking power and stopping time as reported by [12] in equation (3):

$$Work\ done\ (kJ) = power \times stoppingtime \qquad (3)$$

β_0 and σ_0 are the constants to wear rate and coefficient of friction, $\beta_1 - \beta_7$ and $\sigma_1 - \sigma_7$ are the coefficients of the inputs for the wear rate and coefficient of friction respectively.

Table 2. Training and testing inputs to ANN.

Input	Training data set T_1-T_{30}	Samples					
		S_1	S_2	S_3	S_4	S_5	S_6
Epoxy- resin (%) (X_5)	1.5-70	19	40	23	25	15	30
Palm kernel fibres (PKS) (%) (X_1)	0.5-60	6	10	27	30	35	40
Aluminium oxide (%) (X_2)	0.12-15	0	6	10	5	5	5
Graphite (%) (X_3)	2-40	5	29	10	5	5	5
Calcium carbonate (%) (X_4)	15-100	70	15	30	35	40	20
Brake interface temperature (°C) (X_6)	300-1000	690	650	770	800	610	600
Workdone by brake application (kJ) (X_7)	1.67-10	3.84	4.83	5.91	4.01	5.62	2.57

A Reglin function in the SCILAB environment for multiple regression with sample codes shown in Appendix 1 was utilized to evaluate the coefficients $\beta_{0,} - \beta_7$ and $\sigma_{0,} - \sigma_7$ for wear rate and coefficient of friction respectively. The four statistical criteria used in validating the regression models were mean absolute error (MAE), root means square error (RMSE), Nash-Scutcliffe efficiency (NSE) given by the equations (4), (5) and (6) [12,13]:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}(P_i - E_i) \qquad (4)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(P_i - E_i)^2}{N}} \qquad (5)$$

$$NSE = 1 - \frac{\sum_{i=1}^{N}(E_i - P_i)^2}{\sum_{1=1}^{N}(E^i - \bar{E}_i)^2} \qquad (6)$$

where E is the experimental value, P is the predicted value by multiple regression model $\bar{E}$ and $\bar{P}$ are the mean value of E and P respectively, N is the number of sample.

2.5 Artificial neural network modelling

The artificial neural network model for predicting wear rate and coefficient of friction was based on the experimental data generated by the inertia dynamometer. The same inputs used as presented in Table 2 for multiple linear regression models for prediction of wear rate and coefficient were adopted. The inputs data T_1-T_2 were selected outside the ranges of each of the inputs and were used for the training of the neural network to obtain the neural network architecture that matched the inputs/outputs relationship. Several neural network architectures were trained with MATLAB 7.90 using different algorithms (Resilient backpropagation, Levernberg Marquardt, and Bayesian Regulation etc), and layers such as one layered, two layered and three layered networks. However, prior to training of the neural network, the inputs such as brake interface temperature and workdone by brake applications were scaled within the value of 0-1 using equation (7) [14]. While the experimental output wear rate (Table 3) was normalized using the relation in equation (8) reported by [15]. The reason for this is that input and output data set are measured in different units and needed to be normalized into the dimensionless units to remove the arbitrary effect of similarity among the data. The data set S_1-S_2 was employed for testing the prediction capabilities of the artificial neural network for the prediction of wear rate and coefficient of friction. The relationship between the inputs and outputs is shown in Fig. 1. The ANN model was also validated using statistical tools given in equations (4-6).

$$I_{Skal} = 1 + \frac{(I_{Curr} - I_{Max})}{(I_{Max} - I_{Min})} \tag{7}$$

$$y_n = \frac{y - 0.95y_{min}}{1.05y_{max} - 0.95y_{min}} \tag{8}$$

Where: I_{Curr}-current input value, I_{Max}-maximum input value and I_{Min}-minimum input value, y_n is the normalized value of y; y is the experimental data, y_{max} and y_{min} are the max. and min values of y respectively

Table 3. Outputs generated from Inertia dynamometer.

Output	Samples					
	S_1	S_2	S_3	S_4	S_5	S_6
Wear rate (mg/m)	3.59	3.54	3.56	3.52	3.91	4.08
Coefficient of friction	0.17	0.35	0.18	0.19	0.21	0.2

3. RESULTS AND DISCUSSION

3.1 Multiple linear regression (MLR)

The inputs representing the manufacturing conditions were eliminated as there was no relationship with the outputs (wear rate and coefficient of friction); as their values were constant for each of the samples produced. The regression constants and coefficients for the inputs shown in equations (2a) and (2b) were obtained using measured data with the aid of SCILAB software version 5.4.1. These constants and coefficients were substituted into equations (2a) and (2b) to formulate multiple regression models for predicting wear rate and coefficient of friction shown in equations (9a) and (9b) respectively:

$$W_{rate} = 5.409 + 5.85 \times 10^{-4}X_1 + 0.0153X_2 - 0.02X_3 - 0.002X_4 + 0.0062X_5 - 0.0024X_6 - 0.003X_7 + 2.584 \times 10^{-15}. \tag{9a}$$

$$CoF = 0.095 + 0.0031X_1 - 0.0121X_2 + 0.0111X_3 - 2.57 \times 10^{-4}X_4 - 0.0018X_5 + 7.24 \times 10^{-5}X_6 + 9.2 \times 10^{-4}X_7 + 6.28 \times 10^{-16}. \tag{9b}$$

The negative coefficients observed in equations (9a) and (9b) for some of the inputs implied that as the individual responses increased there was decrease in the input parameters and vice versa. This confirms the earlier findings reported by [7,12]. The developed models using MLR were employed to predict the wear rate and coefficient of friction as presented in Tables 4 and 5 respectively.

Tables 4 and 5 show the values of the predicted and measured wear rate and coefficient of friction with their statistical tools used in validating the model. It was observed that the selected neural network prediction of the wear rate and coefficient of friction were in agreement with the measured values. This was confirmed by the values of MAE and RMSE reasonably low; R and NSE reasonably high as reported by [14,6].

3.2 Artificial neural network model (ANN)

The neural network architecture BR 7 [5-3] 2 shown in Fig. 2 was discovered to match the inputs for the prediction of wear rate and coefficient of friction after series of training.

Table 4. The predicted and measured wear rates with performance statistics using MLR and ANN Models.

	Multiple Linear regression Model				Artificial neural network model			
Measured Wear rate (mg/m)	Predicted Wear rate (mg/m)	MAE	RMSE	NSE	Predicted Wear rate (mg/m)	MAE	RMSE	NSE
3.59	3.59	0.0000	0.0000	1.0000	3.41	-0.030	0.0740	0.8990
3.54	3.54	0.0000	0.0000	1.0000	3.54	0.0000	0.0000	1.0000
3.56	3.58	0.0033	0.0087	0.9780	3.60	0.0007	0.0163	0.9184
3.52	3.53	0.0001	0.0041	0.9970	3.52	0.0000	0.0000	1.0000
3.91	3.89	-0.0033	0.0082	0.9910	3.80	-0.018	0.0450	0.7256
4.08	4.08	0.0000	0.0000	1.0000	4.03	-0.0083	0.0500	0.9827

Table 5. The predicted and measured coefficient of friction (CoF) with performance statistics using MLR and ANN models.

	Multiple Linear regression Model				Artificial neural network model			
Measured Coeff. of Friction	Predicted Coeff. of Friction	MAE	RMSE	NSE	Predicted Coeff. of Friction	MAE	RMSE	NSE
0.17	0.173	0.0005	0.0019	0.9874	0.19	0.0030	0.0290	0.8166
0.35	0.35	0.0000	0.0000	1.0000	0.39	0.0067	0.0163	0.9010
0.18	0.18	0.0000	0.0000	1.0000	0.23	0.0083	0.0204	0.8999
0.19	0.193	0.0005	0.0012	0.9873	0.19	0.0000	0.0000	1.0000
0.21	0.21	0.0000	0.0000	1.0000	0.199	-0.0018	0.0045	0.9819
0.25	0,2	0.0083	0.0204	0.9081	0.25	0.0083	0.0204	0.9103

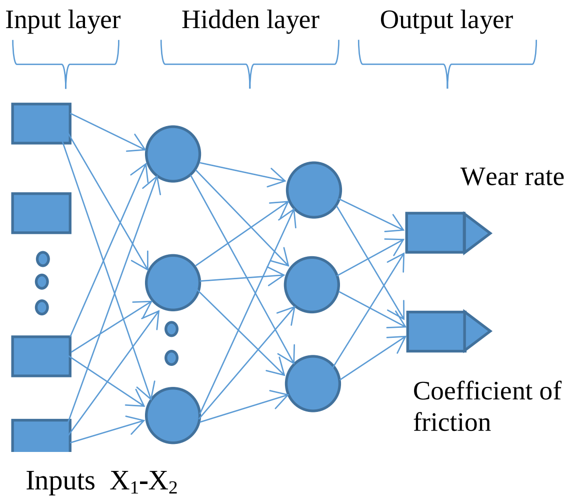

Fig. 2. Neural network architecture used for the prediction of Wear rate and Coefficient of friction.

The choice was based on the training performance indicators that gave a value R = 0.9983 at epoch of 17 with the overall R= 0.09752 for training validation and testing. These values were in consonance with the works of [10,14] for the utilization of ANN for predicting compressibility and oil absorption of gasket produced from palm kernel fibres.

3.3 Comparison of multiple linear regression and artificial neural network models

Tables 4 and 5 present the statistical indicators used for the comparison between the MLR model and ANN model for prediction of wear rate and coefficient of friction. The statistical analyses used for the validation of the models were mean absolute error (MAE), Root mean square Error (RMSE), and Nash-Scutcliffe efficiency (NSE). The values of statistical indicators MAE, RMSE, and NSE for validation of the multiple regression model for prediction of wear rate ranged from -0.0033 – 0.000, 0.000 – 0.0087, and 0.9780 - 1.0000, respectively. While that of coefficient of friction varied from 0.00005-.0.0000, 0.0000-0.0204, and 0.9081-1.0000 respectively for MAE, RMSE, and NSE. This result was in consonance with the studies undertaken by [15,1] where the MAE, RMSE and NSE were within the acceptable limit.

While the corresponding values of the statistical indicators for ANN model varied from -0.0018-0.0000, 0.0000-0.029, and 0.8166-1.0000 for MAE, RMSE and NSE respectively. The statistical indicators show that the prediction of the wear rate and coefficient of friction employing the multiple regression models was satisfactory with the values of, MAE, and RMSE, reasonably low; NSE values reasonably high for almost all the measured values. The predicted wear rate values of 3.59, 3.56 and 4.08 mg/m using MLR model were the same as the experimental values compared to two values of 3.52 and 4.08 mg/m obtained using ANN model. While the predicted coefficient of friction values of 0.35, 0.18 and

0.21 using MLR model were also the same as the experimental values compared to only one value of 0.19 obtained using ANN model. This implies that MLR model predicted wear rate and coefficient of friction of the automotive brake pads developed from palm kernel shell with high accuracy than the ANN model.

The two models were proven to be suitable for predicting the measured values of wear rate and coefficient of friction. The wear rate was observed to increase with increasing percentage of palm kernel shell used for both MLR and ANN models along with measured values. However, the MLR model proved to be better than ANN model as seen in Fig. 3. The increased in wear rates and coefficient of friction (Fig. 3) with increasing percentage of PKS is due to reduced wettability of the shell which act as fibre by the epoxy used as binders. Thus leading to brake pads with more pores which weaken the ability of the epoxy to hold the friction materials together thereby increasing the wear rate as reported by [12,14].

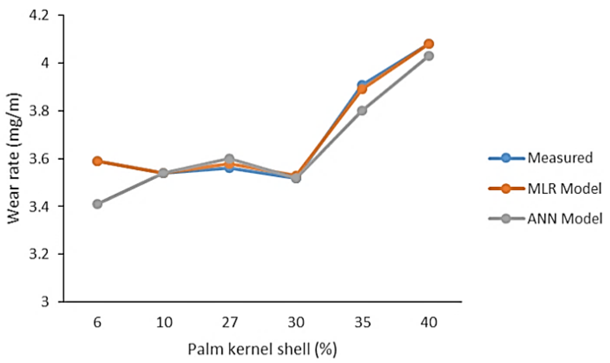

Fig. 3. Comparison of the wear rate obtained by MLR and ANN models with measured values.

The comparison between the wear rates obtained from MLR and ANN models against palm kernel shell clearly showed that MLR model was closer to the measured values compared to ANN model. This was confirmed by good correlation as presented in Fig. 4 with R^2=0.9989 between the predicted and measured using MLR than that of ANN model with R^2=0.912 as presented in Fig. 5. Similarly the accuracy of predicting coefficient of friction using MLR and ANN models along with measured values was graphically compared as presented in Fig. 6. It was also clearly shown that the MLR model was superior over ANN model and increased with increasing percentage of PKS as confirmed by higher correlation value of 0.9124 for MLR model presented in Fig. 7 against 0.8885 for ANN model shown in Fig. 8 as reported by [12].

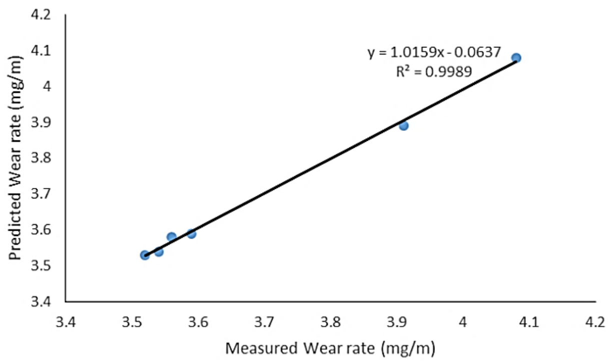

Fig. 4. Scatter plot of the measured and predicted wear rate derived from multiple linear regression model.

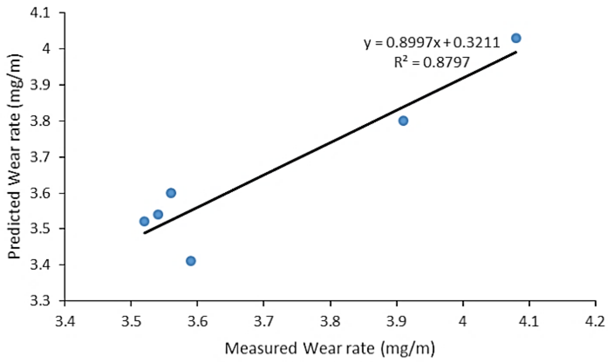

Fig. 5. Scatter plot of the measured and predicted wear rate derived from ANN model.

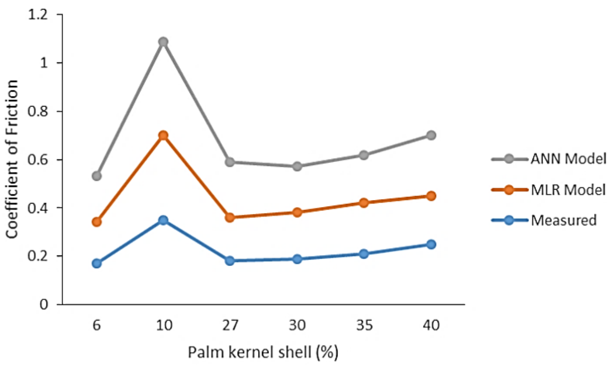

Fig. 6. Comparison of coefficient of friction obtained by MLR and ANN models with measured values.

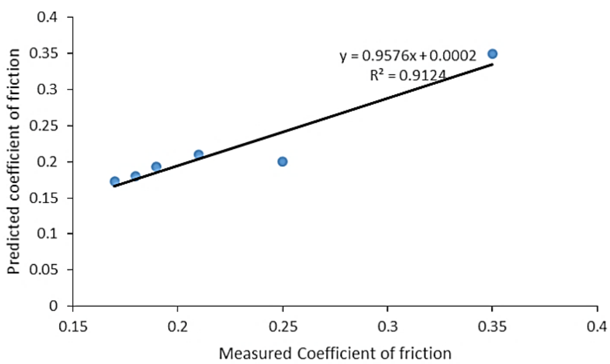

Fig. 7. Scatter plot of the measured and predicted coefficient of friction derived from multiple linear regression model.

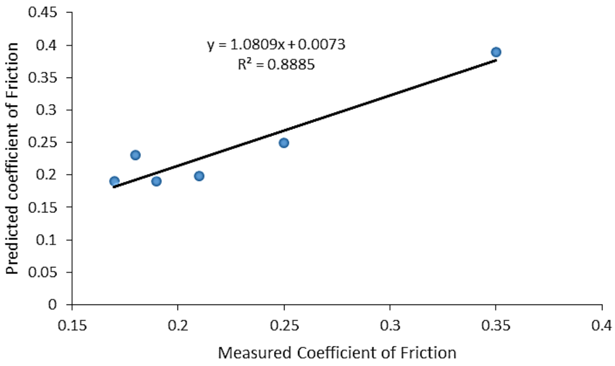

Fig. 8. Scatter plot of the measured and predicted coefficient of friction derived from ANN model.

The analysis of variance (ANOVA) shown in Appendix 2a and 2b was carried out to determine whether there is significant difference between the MLR and ANN models together with measured values in terms of wear rate and coefficient of friction at 5 % significant level. The ANOVA results revealed that there was no significant differences (Ho: $F < F_{crit}$; $0.097505 < 3.68232$ for wear rate; and $0.181172 < 3.68232$ for coefficient of friction) between MLR and ANN models along with the measured values. Thus suggesting that the values of wear rate and coefficient of friction for the two models in question are statistically the same with the measured values.

4. CONCLUSION

The comparative analysis of the MLR and ANN models for predicting wear rate and coefficient of friction for automotive brake pads developed from palm kernel shell was successfully carried out. The conclusions drawn were:

1. Both MLR and ANN models approaches have potential for predicting the wear rate and coefficient of friction of automotive brake pads.
2. MLR model performed better in predicting wear rate and coefficient of friction compared to ANN model based on the statistical indicators and visual assessment. That is the two MLR equations are capable of predicting the wear rate and coefficient of friction to an appreciable level of accuracy.
3. That the two models can be used as complementary to each other in making good decision concerning the prediction of wear rate and coefficient of friction of brake pads and other fiction composites provided the inputs are identified.

REFERENCES

[1] K.I. Kumaden, A. Ashwe, T.T. Livinus, *Prediction of wear and friction coefficient of brake pads developed from palm kernel fibres using artificial neural network*, Journal of Engineering Research Studies, vol. 20, iss. 3, pp. 45-54, 2014.

[2] F. Zafaruddin, D.R. Dolas, *Experimental investigation of organic brake pad*, International Journal of Advance Research and Innovative Ideas in Education, vol. 2, iss. 6, pp. 213-219, 2016.

[3] S. Goyal, G.K. Goyal, *Application of artificial neural engineering and regression models for forcasting shell life of instant coffee drink*, International Journal of Computer Science Issues, vol. 8, iss. 4, no. 1, pp. 320- 324, 2011.

[4] L.B. Mustafar, R. Razali, *A study on prediction of output in oil field using multiple linear regression*, International Journal of Applied Science and Technology, vol. 1, no. 4, pp. 107-113, 2011.

[5] J. Mata, *Interpretation of concrete dam behaviour with artificial neural network and multiple linear regression models*, Engineering Structures, vol. 33, iss. 3, pp. 903-910, 2011, doi: 10.1016/j.engstruct.2010.12.011

[6] A.K.F. Hassan, S. Mohammed, *Artificial neural network model for estimation of wear and temperature in Pin-disc contact*, Universal Journal of Mechanical Engineering, vol. 4, no. 2, pp. 39-49, 2016, doi: 10.13189/ujme.2016.040204

[7] J.T. Lin, D. Bhattacharyya, V. Kecman, *Multiple regression and neural networks analyses in composite machining*, Composites Science and Technology, vol. 63, iss. 3-4, pp. 539-548, 2003, doi: 10.1016/S0266-3538(02)00232-4

[8] S.R. Panda, A.K. Senapati, P.C. Mishra, *Multiple regression prediction model for surface roughness and tool wear in turning novel aluminium alloy (LM6)/fly ash using response surface and central composite design methodology*, International Journal of Industrial Engineering computations, vol. 8, iss. 1, pp. 1-18, 2017, doi: 10.5267/j.ijiec.2016.8.001

[9] A. Kialashaki, J.R. Reisel, *Modelling of energy demand of the residential sector in the United States using regression models and artificial neural networks*, Applied Energy, vol. 108, pp. 271-280, 2013, doi: 10.1016/j.apenergy.2013.03.034

[10] K.K. Ikpambese, D.T. Gundu, L.T. Tuleun, *Evaluation of palm kernel fibres for the*

production of asbestos free- automotive brake. Journal of King Saud University-Engineering Sciences, vol. 28, iss. 1, pp. 110-113, 2016, doi: 10.1016/j.jksues.2014.02.001

[11] J.I. Liu, R.D. Paode, T.M. Holsen, *Modelling the energy content of municipal solid waste using multiple regression analysis*, Journal of Air and Waste Management Association, vol. 46, iss. 7, pp. 650-656, 1996, doi: 10.1080/10473289.1996.10467499

[12] K.K. Ikpambese, *Evaluation and modelling of friction and wear of automotive brake pads developd from palm kernel wastes*, Ph.D. thesis, College of Engineering, University of Agriculture Makurdi, Benue State , Nigeria, 2014.

[13] S. Mohanty, M.K. Jha, A. Kumar, D.K. Panda, *Comparative evaluation of numerical model and artificial neural network for simulating groundwater flow in Kathajodi–Surua Inter-basin of Odisha, India*, Journal of Hydrology, vol. 495, pp. 38-51, 2013, doi: 10.1016/j.jhydrol.2013.04.041

[14] K.K. Ikpambese, L.T. Tuleun, P. Ager, *Artificial neural network prediction of compressibility and oil absorption of gasket produced from palm kernel fibres*, Journal of Engineering research, University of Lagos, vol. 17, no. 3, pp. 32-39, 2012.

[15] D.T. Gundu, L.T. Tuleun, K.K. Ikpambese, *Comparative evaluation of 3-d numerical and artificial neural network models for predicting extrusion pressure and product deflection*, American Journal of Scientific and Industrial Research, vol. 4, pp. 378-387, 2013.

Appendix 1

Sample Code for Multiple Regressions using the Reglin Function:

```
x1=[6 10 27 30 35 40]
x2=[0 6 10 5 5 5]
x3=[5 29 10. 5 5 5]
x4=[70 15 30 35 40 20]
x5=[19 40 23 25 15 30]
x6=[690 650 770 800 610 600]
x7=[3.84 4.83 5.91 4.01 5.62 2.57]
y=[3.59 3.54 3.56 3.52 3.91 4.08]
X=[x1; x2; x3; x4; x5; x6; x7];// create matrix x from the six rows
[a, a0, sigma]=reglin (X,y);// perform a multiple regression analysis of y as a function of x1, x2, x3, x4, x5,x6 and x7
ypred = a0 + a(1)*x1 + a(2)*x2 + a(3)*x3 + a(4)*x4 + a(5)*x5 + a(6)*x6 + a(7)*x7;// fitted data
wn = scf() ;//create new graphic window
plot(y, ypred, "*");//plot given data with fitted data
d = gca();// get current graphic attributes
d.children.children.thickness = 3;// make the plot lines 3 points thick
Xtitle("",Given data", "Fitted data");// specify axes titles
filename = myreglindir + basename(myfile) + "-plot.png";// create filename from basename of file
xs2png(wn, filename);// save graphic in PNG format
result = ["a0" "a(1)" "a(2)" "a(3)" "a(4)" "a(5)" "a(6)" "a(7)"];// first row of data
result(2,:) = string([a0 a]);// convert coefficients to string for export
result(:,$) = result(:,$) + "\\\ hline";//add end of line commands for LaTex table
filename = myeeglindir + basename(myfile) + "text";// filename for saving results
csvWrite(result, filename, "&");// use "&" to separate the colums
```

Appendix 2

Comparison of wear rates values from MLR and ANN models together with measured values using Analysis of Variance (ANOVA):

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.010344	2	0.005172	0.097505	0.907668	3.68232
Within Groups	0.795683	15	0.053046			
Total	0.806028	17				

Ha: F > Fcrit   Ho: F ≤ Fcrit , α = 0.05
There is no significant

Appendix 3

Comparison of coefficient of friction values from MLR and ANN models together with measured values using Analysis of Variance (ANOVA):

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.001788	2	0.000894	0.181172	0.836091	3.68232
Within Groups	0.074023	15	0.004935			
Total	0.075811	17				

Ha: F > Fcrit   Ho: F ≤ Fcrit , α = 0.05
There is no significant