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Performance of a Magnetic Fluid Based Squeeze Film Between Transversely Rough Triangular Plates

RESEARCH

Efforts have been made to study and analyze the effect of transverse surface roughness on the performance of a magnetic fluid based squeeze film between triangular plates. The roughness of the bearing surfaces is characterized by a stochastic random variable with non-zero mean variance and skewness. The associated Reynolds' equation is stochastically averaged with respect to the random roughness parameter in turn, which is solved with appropriate boundary conditions to obtain the pressure distribution. From this, the expression for load carrying capacity is derived. The results suggest that the magnetization parameter increases the load carrying capacity while the load carrying capacity gets decreased due to the standard deviation. It is seen that the negatively skewed roughness increases the load carrying capacity substantially especially, when the negative variance is involved. It is revealed that the negative effect induced by the standard deviation can be minimized by the positive effect of the magnetization parameter in the case of negatively skewed roughness.

Keywords: *Squeeze film, Triangular plates, Magnetic fluid, Roughness, Load carrying capacity.*

1. INTRODUCTION

Archibald [1] investigated the performance of squeeze film behavior between various geometrical configurations. Later on Wu [2, 3] analyzed the squeeze film performance for two types of geometries namely; annular and rectangular when the upper plate having a porous facing approaches the lower non-porous plate. Subsequently, Prakash and Viz [4] modified some aspects of Wu [2, 3] using Morgan Cameron approximation. In fact, they obtained the load carrying capacity and the time height relation for squeeze film between porous plates. This article, dealt with various geometries like circular, annular, elliptical and rectangular plates. Besides, a comparison was made between the squeeze film behavior of various geometries of equivalent surface area, other parameters remaining the same. It was observed that the circular plates recorded the highest transient load carrying capacity.

Conventional lubricants were considered in all the above studies. The application of magnetic fluid as a lubricant was studied and analyzed by Verma [5]. The magnetic fluid considered, consisted of fine magnetic grains coated with a surfactant and magnetically passive solvent. Bhat and Deheri [6] observed the squeeze film behavior between porous annular disks using a magnetic fluid lubricant with the external magnetic field oblique to the lower disk and observed that the performance with the magnetic lubricant was better than that with conventional lubricant. The analysis of Bhat and Deheri was improved by Bhat and Deheri [7] to study the behavior of a magnetic fluid based squeeze film between curved circular plates.

Patel and Gupta [8] analyzed the effect of a transverse magnetic field on the behavior of squeeze film between porous plates of different geometries. Prajapati [9] discussed the behavior of hydromagnetic parallel squeeze film between conducting porous surfaces. In this study various geometries including elliptical, conical and rectangular were subjected to investigations.

Due to elastic thermal and uneven wear effects, the configurations encountered in practice are usually far from being smooth. Besides, it is a well known fact that the bearing surfaces after having some run-in and wear develop roughness. The effect of random surface roughness was analyzed by many

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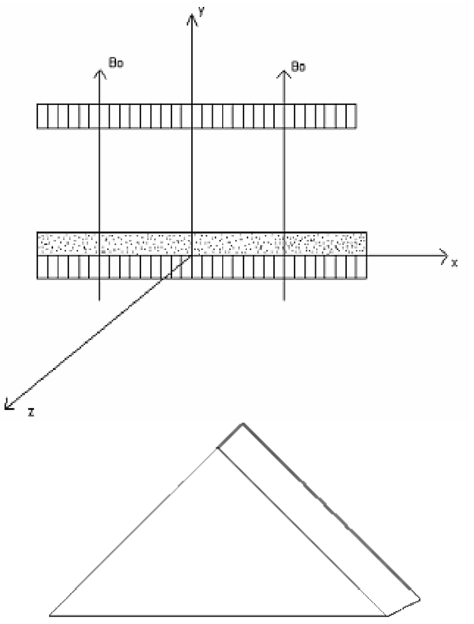
investigators [Davis [10], Burton [11], Tonder [12], Tzeng and Saibel [13]]. Christensen and Tonder [14, 15, 16] developed the approach of Tzeng and Saibel and proposed a comprehensive general analysis both for transverse as well as longitudinal surface roughness. This analysis of Christensen and Tonder [14, 15, 16] formed the basis of studying the effect of surface roughness in a number of investigations [Ting [17], Prakash and Tiwari [18], Prajapati [19, 20], Guha [21], Gupta and Deheri [22], Andharia, Gupta and Deheri [23, 24]]. Patel and Deheri [25, 26] dealt with the performance of a magnetic fluid based squeeze film between circular and annular plates and studied the effect of surface roughness on the performance of the bearing system. Recently, the effect of transverse surface roughness on the performance of hydromagnetic squeeze film between conducting rough porous plates of different geometrical configurations was analyzed by Vader et. al. [27, 28, 29]. Here, it has been sought to analyze the performance of a magnetic fluid based squeeze film behavior between triangular plates.

2. ANALYSIS

The geometry and the configuration of bearing system is given below.



Configuration of the bearing system

The lower plate with a porous facing is assumed to be fixed while the upper plate moves normally towards the lower plate with the uniform velocity $\dot{h} = dh/dt$. The flow in the porous medium obeys the modified form of Darcy's law (Ene [30]), while in the film region the equations of hydrodynamic lubrication theory hold. The bearing surfaces are assumed to be transversely rough. The geometry of the local film thickness can be thought of as consisting of two parts:

$$h(x) = \bar{h}(x) + h_s(x)$$

where $\bar{h}(x)$ is the mean film thickness and $h_s(x)$ is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. $h_s(x)$ is considered to be stochastic in nature and governed by the probability density function $f\ (h_s),\ \ -c \le h_s \le c$ where c is the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε which is the measure of symmetry of random variable h_s, are defined by relationships

$$\alpha = E\ (h_s)$$

$$\sigma^2 = E\ [\ (h_s - \alpha)^2\ ]$$

and

$$\varepsilon = E\ [\ (h_s - \alpha)^3\ ]$$

where the expectancy operator E is defined by

$$E\ (R) = \int_{-c}^{c} Rf(h_s)dh_s$$

while

$$f(h_s) = \begin{cases} \dfrac{35}{32c^7}(c^2 - h_s^2)^3, if -c \le h_s \le c \\ 0, elsewhere \end{cases}$$

Assuming symmetric flow of the magnetic fluid about $x=0$ between the parallel plates under an oblique magnetic field $\bar{H}$ whose magnitude H is a function of x vanishing at

$$x=a,\ x=\sqrt{3}\,z-2a \text{ and } x=-\sqrt{3}\,z-2a$$

where a is the length of each side of the equilateral triangle and stochastically averaging the associated differential equation in view of the analysis of the Christensen and Tonder [14, 15, 16] the modified Reynolds' equation governing the film pressure is obtained as (Bhat and Deheri [31], Patel and Deheri [32], Prakash and Viz [4])

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right]\left(p - 0.5\mu_0\bar{\mu}H^2\right) = \frac{12\mu\dot{h}}{g(h) + 12\phi H_0} \quad (1)$$

where in

$$H^2 = K(x-a)(x-\sqrt{3}z+2a)(x+\sqrt{3}z+2a)$$

K being suitably chosen constant from dimensionless point of view and

$$g(h) = h^3 + 3h\sigma^2 + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon$$

The inclination of the magnetic field is given by the solution of linear first order partial differential equation

$$cot\theta\frac{\partial\theta}{\partial x} + \frac{\partial\theta}{\partial z} = \frac{2\sqrt{3}-(4x-4\sqrt{3}z-a)}{3-2x(z-2\sqrt{3}a)} \tag{2}$$

and μ is the viscosity of the lubricant, μ_0 is the permeability of the free space and $\bar{\mu}$ is the magnetic susceptibility.

Integration of equation (1) with respect to the appropriate boundary conditions

$$p(x_1,z_1)=0 \tag{3}$$

where in

$$(x_1-a)(x_1-\sqrt{3}\,z_1+2a)(x_1+\sqrt{3}\,z_1+2a)=0 \tag{4}$$

and introduction of the following non-dimensional quantities

$$\mu^* = -\frac{Kh^3\mu_0\bar{\mu}}{\mu\dot{h}}, \qquad \alpha^* = \frac{\alpha}{h},$$

$$\sigma^* = \frac{\sigma}{h}, \qquad \varepsilon^* = \frac{\varepsilon}{h^3},$$

$$\psi = \frac{\varphi H_0}{h^3}$$

$$g(\bar{h}) = 1 + 3\sigma^{*2} + 3\alpha^* + 3\alpha^{*2} + 3\sigma^{*2}\alpha^* + \alpha^{*3} + \varepsilon^*$$

leads to the expression for non-dimensional pressure distribution:

$$P^* = -\frac{h^3 p}{3\mu\dot{h}\sqrt{3}a^2}$$

$$= \frac{4}{3\sqrt{3}}\left[\frac{\mu^*}{12} + \frac{1}{g(\bar{h})+12\psi}\right]\left(1-\frac{x}{a}\right)$$

$$\left(1+\frac{\sqrt{3}z}{2a}+\frac{x}{2a}\right)\left(1-\frac{\sqrt{3}z}{2a}+\frac{x}{2a}\right) \tag{5}$$

Then the load carrying capacity given by

$$W = \int_{-2a}^{a}\int_{-\frac{(x+2a)}{\sqrt{3}}}^{\frac{(x+2a)}{\sqrt{3}}} p\,dx\,dz \tag{6}$$

is obtained in dimensionless form as

$$W^* = -\frac{h^3 W}{27\mu\dot{h}a^4}$$

$$= \frac{\sqrt{3}}{5}\left[\frac{\mu^*}{12} + \frac{1}{g(\bar{h})+12\psi}\right] \tag{7}$$

Finally, the time Δt taken by the upper plate to reach a film thickness h_2 from an initial film thickness h_1 can be obtained in dimensionless form as

$$\Delta\bar{t} = -h_0{}^2W^*\int_{\bar{h}1}^{\bar{h}2}\frac{d\bar{h}}{G(\bar{h})} \tag{8}$$

where

$$G(\bar{h}) = \bar{h}^3 + 3\sigma^{*2}\bar{h} + 3\bar{h}^2\alpha^* + 3\bar{h}\alpha^{*2} + 3\sigma^{*2}\alpha^* + \alpha^{*3} + \varepsilon^* + 12\psi$$

$$\bar{h}1 = \frac{h_1}{h_0}, \ \bar{h}2 = \frac{h_2}{h_0} \text{ and } \bar{h} = \frac{h}{h_0}$$

3. RESULTS AND DISCUSSIONS:

Equation (5) determines the dimensionless pressure distribution while equation (7) presents the non-dimensional load carrying capacity. In addition, the response time in dimensionless form is obtained from equation (8). These above three expressions depend on several parameters such as μ^*, ψ, α^*, σ^* and ε^*. However, from equations (7) and (8) it is clear that the expression for non-dimensional load and response time do not contain a explicitly. Taking the roughness parameters to be zero the study reduces to the performance of magnetic fluid based squeeze film behavior between porous triangular plates. Further, setting the magnetization parameter to be zero one arrives at the performance of squeeze film between triangular plates as studied by Prakash and Vij [4]. It is clearly observed that the load carrying capacity increases by $\mu^* / 20\sqrt{3}$ due to the magnetization parameter, as indicated by equation (7).

The variation of load carrying capacity with respect to the magnetization parameter for various values of porosity, standard deviation, variance and measure of symmetry is presented in Figures (1- 4).

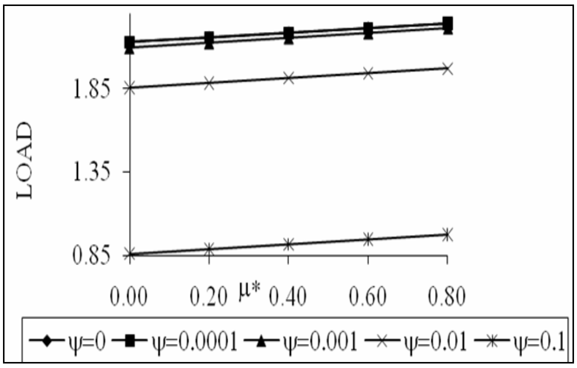


Figure 1. Variation of load carrying capacity with respect to μ* and ψ.

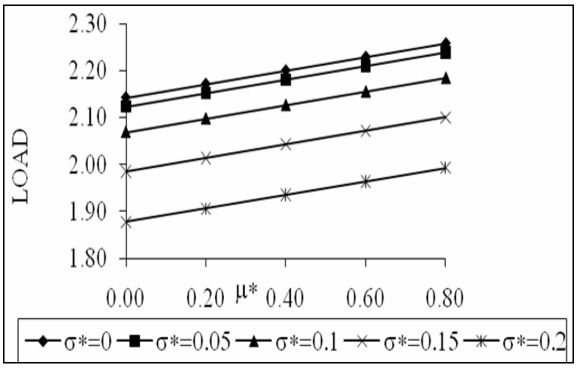


Figure 2. Variation of load carrying capacity with respect to μ* and σ*.

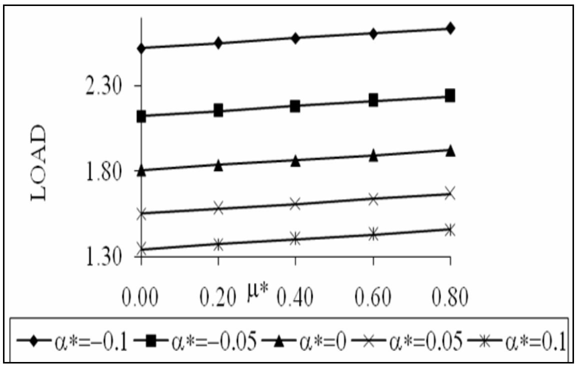


Figure 3. Variation of load carrying capacity with respect to μ* and α*.

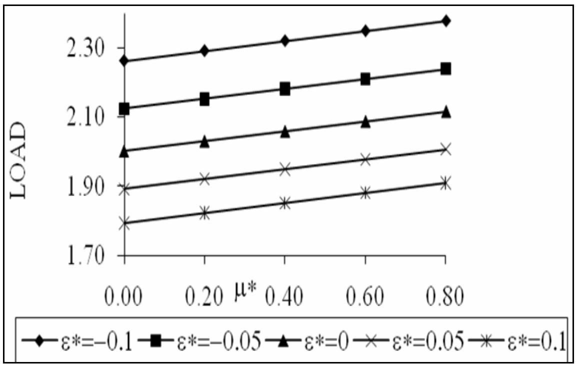


Figure 4. Variation of load carrying capacity with respect to μ* and ε*.

It is noticed that the load carrying capacity increases sharply with respect to the magnetization parameter which underlines that the performance of the bearing system gets improved considerably due to the magnetic fluid lubricant. Further, porosity decreases the load carrying capacity. Besides, positive variance and skewness (positive) decrease the load carrying capacity. Also, negatively skewed roughness increases the load carrying capacity. Same is the case with negative variance.

The effect of porosity on the distribution of load carrying capacity is demonstrated in Figures (5 - 7).

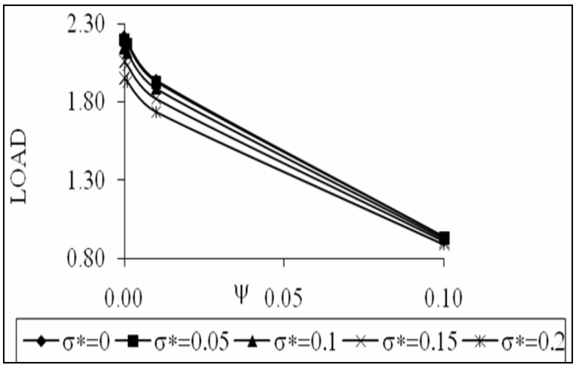


Figure 5. Variation of load carrying capacity with respect to ψ and σ*.

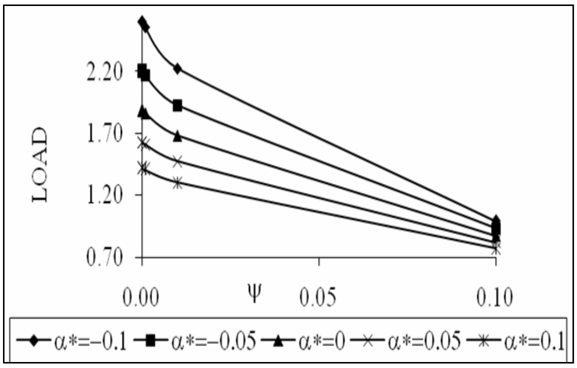


Figure 6. Variation of load carrying capacity with respect to ψ and α*.

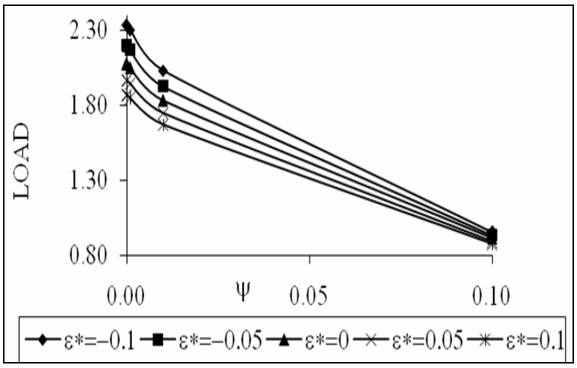


Figure 7. Variation of load carrying capacity with respect to ψ and ε*.

These figures make it clear that porosity has a considerably adverse effect on the performance of the bearing system. This adverse effect is more pronounced in the case of measure of symmetry. Besides, from Figure (5) it is clear that the effect of standard deviation with respect to porosity is comparatively less.

Finally, the effect of roughness parameters is given in Figures (8 - 10).

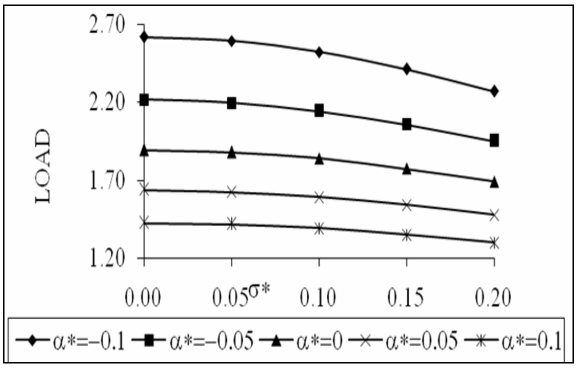


Figure 8. Variation of load carrying capacity with respect to σ∗ and α∗.

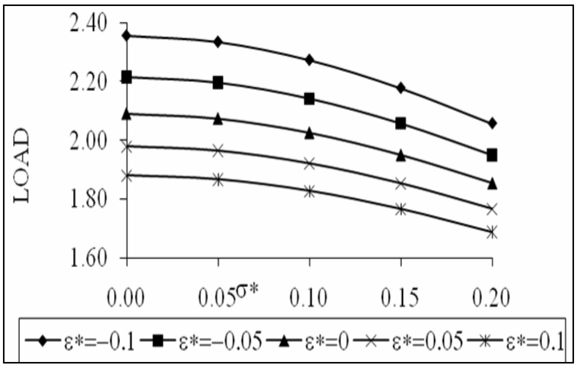


Figure 9. Variation of load carrying capacity with respect to σ∗ and ε∗.

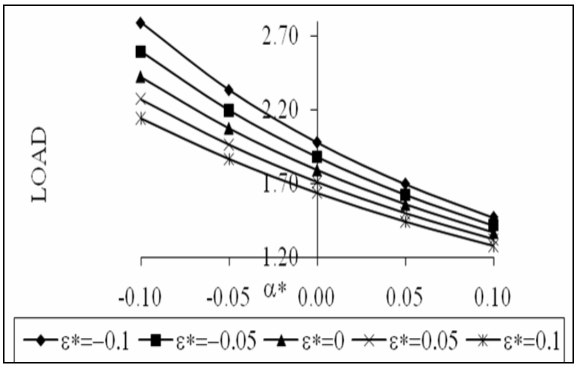


Figure 10. Variation of load carrying capacity with respect to α∗ and ε∗.

Here, it is easily observed that the combined effect of standard deviation and positive skewness is sharp as compared to the combined effect of standard deviation and variance. Alone these three figures suggest that there is considerable increase in the load carrying capacity due to the negatively skewed roughness especially, when the negative variance is involved.

4. CONCLUSION:

It is revealed from this study that the adverse effect of the standard deviation and porosity can be minimized by the magnetization parameter in the case of negatively skewed roughness when negative variance occurs. Further, this study makes it mandatory that the roughness must be given due consideration while designing the bearing system even if a suitable value of magnetization parameter has been chosen.

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