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RESEARCH

Surface Roughness and Elastic Deformation Effects on the Behaviour of the Magnetic Fluid Based Squeeze Film Between Rotating Porous Circular Plates with Concentric Circular Pockets

An attempt has been made to study and analyze the performance of a magnetic fluid based squeeze film between rotating porous transversely rough circular plates with concentric circular pockets. The porous housing is considered to be elastically negligibly deformable with its contact surface transversely rough. The stochastic film thickness characterizing the random roughness is assumed to be asymmetric with non zero mean and variance. The pressure distribution is obtained by solving the associated stochastically averaged Reynolds equation with appropriate boundary conditions. This results in the calculation of the load carrying capacity. All the results in graphical form establish that the transverse roughness in conjunction with the deformation has a strong negative effect on the performance of the bearing system. The bearing suffers on account of transverse surface roughness in general which probably is due to the fact that the roughness of the bearing surfaces tends to retard the motion of the lubricant resulting in decreased load carrying capacity. However, this negative effect of roughness, porosity and deformation can be minimized by the positive effect of the magnetization parameter in the case of negatively skewed roughness by choosing a suitable combination of pocket radius and rotational inertia. Lastly, the effect of radii ratio is noted to be quite significant.

Keywords: *Squeeze film, magnetic fluid, rough surface, Reynolds equation, load carrying capacity, deformation, circular pockets.*

1. INTRODUCTION

Wu [1] considered the analysis of the behavior of squeeze film between porous annular disks while Murti [2] dealt with the squeeze film behavior in porous circular disks. Prakash and Vij [3] used the Morgan Cameron approximation and analysed squeeze film problems involving various geometries. Wu [4] analysed the squeeze film between rotating porous annular disks.

Low design cost, design flexibility, and above all, self lubricating nature of such bearings has made then indispensible for various applications in automatic transmissions, farm and dairy equipments, textile and printing machineries etc.

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Ting [5] investigated the problem of engagement of porous clutch plates, simulating by annular plates incorporating the effects of elastic deformation and surface roughness of the porous housing. However, it is well known that the corresponding analysis for porous circular plates cannot be derived from this analysis because of logarithmic singularities. In many practical situations involving mechanical elements, in addition to the effects of elastic deformations and the roughness of the contact surfaces, pockets are also encountered in the form of dents and cavities. These pockets are mostly circular and concentric in nature resulting from the wearing out of the material due to rotating motion in several cases. Therefore, the study of porous metal lubrication involving such machine elements is of particular importance. Archibald [6] for the first time presented an analysis for the squeeze film between the smooth rigid non-porous circular plates with concentric pockets. All the above studies dealt with conventional lubricants. Bhat and Deheri [7] analysed the squeeze film behaviour between porous annular disks using a magnetic fluid lubricant with the external magnetic field oblique to the lower disk. It was concluded that the use of magnetic fluid as lubricant resulted in the overall improvement of performance characteristics. Besides, it was found that the performance of the magnetic fluid lubricant was much better than that with the conventional lubricant. Later on, Bhat and Deheri [8] discussed the behavior of the squeeze film in curved porous circular plates under the presence of a magnetic fluid lubricant. Shah and Bhat [9] extended the above analysis by considering rotation. But due to elastic thermal and uneven wear effects configurations found in practice are usually far from being smooth. Besides, the bearing surfaces after having some run-in and wear develop roughness. The roughness appears to be random in character, which does not seem to follow any particular structural pattern.

Christensen and Tonder [10]; [11]; [12] modified and developed the analysis of Tzeng and Saibel [13] and presented a comprehensive general analysis for both transverse as well as longitudinal surface roughness. Subsequently, this approach of Christensen and Tonder[11]; [12];[13] formed the basis of investigation into the effect of surface roughness in a number of investigations [Ting[5], Prakash and Tiwari[14], Prajapati [15]; [16], Guha[17], Gupta and Deheri[18]]. Patel and Deheri [19] launched an investigation in to the behaviour of magnetic fluid based squeeze film between annular plates and studied the effect of transverse surface roughness. The squeeze film behaviour between rotating porous circular plates with a concentric pocket was discussed by Prajapati [20] considering only a particular type of roughness. This paper seeks to analyze the performance of magnetic fluid based squeeze film behaviour between rotating rough porous circular plates with a concentric circular pocket.

2. ANALYSIS

The geometry and configuration of the bearing system is presented below.

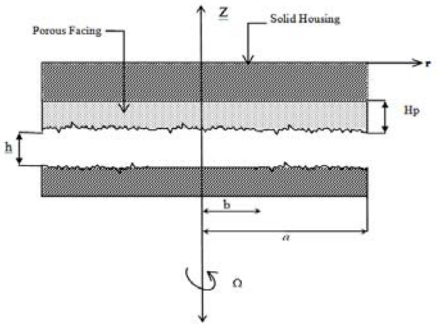

Figure 1. Geometry of the bearing

The squeezing motion of a porous circular plate of radius a approaching a non-porous circular plate with concentric circular pocket extending from its centre and of radius b, is considered. The lower plate is taken to be rotating with a given angular velocity Ω. Both the plates are considered elastically negligibly deformable and their contact surfaces are assumed to be transversely rough. In view of the discussions carried out in Christensen and Tonder [10]; [11]; [12] and Ting [5] the film thickness H has been expressed as

$$H = h(r,t) + \delta(r,t) + h_s(r,\xi)$$

where, h denotes the smooth and unstressed part of the film thickness, δ is the local elastic deformation of the porous facing, h_s is the part due to surface roughness measured from the mean level $h+\delta$. σ, α, ε are the standerd deviation, mean and measure of symmetry of the stochastic film thickness distribution obtained by the relationships

$$\alpha = E(h_s)$$

$$\sigma^2 = E[(h_s - \alpha)^2]$$

and

$$\varepsilon = E[(h_s - \alpha)^3]$$

where E denotes the expected value defined by

$$E(R) = \int_{-c}^{c} R\, f(h_s) ds$$

while the probability density function is represented as

$$f(h_s) = \begin{cases} \frac{35}{32}\left(1 - \frac{{h_s}^2}{c^2}\right)^3, & -c \le h_s \le c \\ 0, & otherwise \end{cases}$$

where c is the maximum deviation from the mean film thickness. The porous medium has permeability ϕ and thickness H_p and is considered to be isotropic and homogeneous. Assuming axially symmetric flow of the magnetic fluid between the disks under and oblique magnetic field $\bar{M}$ whose magnitude M is a function of r vanishing at $r = a$; the modified Reynolds equation governing the film pressure p [Bhat and Deheri[8], Prajapati[20]; [21], Gupta and Deheri [18], Ting [5]] is obtained as

$$\frac{1}{r}\frac{d}{dr}\left[rA\frac{d}{dr}\left(p - \frac{\mu_0\bar{\mu}}{2}M^2\right)\right] = \kappa + 4SB. \qquad (1)$$

where

$$B = h^3 + 3\alpha + 3\sigma^2 + 3\sigma^2\alpha + 3\alpha^2 + \alpha^3 + \varepsilon + 12\phi H_p;$$

$$A = \left(1 + \bar{p}\,\bar{\delta}\right)^3 + 3\left(1 + \bar{p}\,\bar{\delta}\right)\sigma^2 + B - h^3\ ;$$

and

$$\kappa = \frac{12\mu\dot{h}a^2}{p_a h^3}$$

μ_0 is permeability of free space, $\bar{\mu}$ is the magnetic susceptibility of particles and μ is the viscosity of the lubricant. Taking, for instance,

$$M^2 = ka(a-r), \quad 0 \le r \le a$$

and $k = 10^{14}A^2m^{-4}$ chosen so as to have a magnetic field of strength over 10^5 Bhat [22]. Introducing the non-dimensional quantities

$$P = -\frac{h^3 p}{\mu a^2 \dot{h}}, \; R = \frac{r}{a}, \; \mu^* = -\frac{kh^3\mu_0\bar{\mu}}{\mu\dot{h}},$$
$$\bar{\sigma} = \frac{\sigma}{h}, \quad \bar{\alpha} = \frac{\alpha}{h}, \quad \bar{\varepsilon} = \frac{\varepsilon}{h^3} \quad s = \frac{3\delta\Omega^3}{20p_a}, \; \bar{\delta} = \frac{p_a H_p}{hE'}, \; \psi = \frac{\phi H_p}{h^3};$$

where, p_a is the reference ambient pressure, and integrating equation (1) with the associated boundary conditions

$$P = 0 \;, R = 0$$

and

$$\frac{dP}{dR} = -\frac{\mu^*}{2}, \; R = 1$$

one obtains the pressure distribution in non-dimesional form as

$$P = \frac{\mu^*}{2}(1-R) + \frac{\left[3+\left(\frac{s}{\kappa}\right)g(\bar{h})\right](1-R^2)}{g(\bar{h})+3\bar{p}\,\bar{\delta}(1+\bar{\sigma}^2)} \qquad (2)$$

where

$$g(\bar{h}) = 1 + 3\bar{\alpha} + 3\bar{\sigma}^2 + 3\bar{\sigma}^2\bar{\alpha} + 3\bar{\alpha}^2 + \bar{\alpha}^3 + \bar{\varepsilon} + 12\psi$$

The load carrying capacity of the bearing is given by

$$w = 2\pi\int_b^a r\,p(r)\;dr$$

which is calculated in dimensionless form, and it turns out to be

$$\mathrm{W} = -\frac{h^3 w}{2\pi\mu\dot{h}_0\,a^4} = \int_{b/a}^{1} \mathrm{RP\,dR}$$

and hence

$$W = \frac{\mu^*}{2}\left[\frac{1}{6} + \frac{b^3}{3a^3} - \frac{b^2}{2a^2}\right]$$

$$+\frac{0.25\left[3+\left(\frac{s}{\kappa}\right)g(\bar{h})\right]}{g(\bar{h})+3\bar{p}\,\bar{\delta}(1+\bar{\sigma}^2)}\left(1-\frac{b^2}{a^2}\right)^2 \qquad (3)$$

3. RESULTS AND DISCUSSION

It is clearly seen that the dimensionless squeeze film pressure is determined by Equation (2) while Equation (3) presents the distribution of non-dimensional load carrying capacity. These two equations make it clear that the pressure distribution and load carrying capacity depend on various parameters such as μ^*, $\bar{\sigma}$, $\bar{\varepsilon}$, $\bar{\alpha}$, $\frac{b}{a}$, ψ, $\frac{s}{\kappa}$, $\bar{\delta}$. These parameters respectively, describe the effect of magnetic fluid lubricant, surface roughness, radii ratio, porosity, rotational inertia and deformation. The effect of pocket size in the bearing on squeeze film pressure and load carrying capacity is characterized by the parameter b. It is observed from Equation (2) that the fluid film pressure is independent of pocket size where as the pocket size has definite effect on the distribution of load carrying capacity as indicated by Equation (3). However, a closed glance at Equation (3) suggests that increasing values of pocket radius result in decreased load carrying capacity. Besides, Equation (3) makes it clear that the load carrying capacity increases with increasing magnetization parameter. Setting the magnetization parameter to be zero and taking the variance to be zero, this investigation reduces essentially to the investigation of Prajapati [20]. It is revealed that the increase in pressure is

$$\frac{\mu^*}{2}(1-R)$$

while the load carrying capacity enhances by

$$\frac{\mu^*}{2}\left[\frac{1}{6} + \frac{b^3}{3a^3} - \frac{b^2}{2a^2}\right]$$

due to the magnetic fluid lubricant. Taking $\bar{\delta}$, $\bar{\sigma}$, $\bar{\alpha}$ and $\bar{\varepsilon}$ equal to zero, one can avail the magnetic fluid based squeeze film performance for smooth rigid circular rotating porous bearing. Lastly, the results of Prakash and Vij [3] can be recovered by considering the magnetization parameter and the rotational inertia to be zero.

It is noticed from the term containing $\bar{\delta}$ in Equation (2) that the elastic deformation distorts the parabolic profile of the pressure distribution, although, the symmetry of the profile remains undisturbed. Moreover, the elastic deformation of the bearing decreases the film pressure as well as the load carrying capacity which can be easily seen from Equations (2) and (3). It is significant to observe from Equations (1) and (2) that as far as the film pressure distribution and load carrying capacity are concerned, a smooth elastic bearing with inertia parameters

$$\bar{s} = sg(\bar{h}),$$

permeability parameter

$$\bar{\psi} = \psi + 0.25(\bar{\alpha} + \bar{\sigma}^2 + \bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^2) + \frac{1}{12}(\bar{\alpha}^3 + \bar{\varepsilon})$$

and deformation parameter

$$\delta^* = 3\bar{p}\,\bar{\delta}(1 + \bar{\sigma}^2)$$
(for a constant value of $\bar{p}$)

would be equivalent to a rough porous bearing with inertia parameter s, permeability parameter ψ with surface roughness parameters $\bar{\sigma}$, $\bar{\alpha}$, $\bar{\varepsilon}$.

Figures 2-8 dealing with distribution of load carrying capacity with respect to the magnetization parameter μ^* for various values of $\bar{\sigma}$, $\bar{\varepsilon}$, $\bar{\alpha}$, $\frac{b}{a}$, ψ, s/κ and $\bar{\delta}$; make it clear that the load carrying capacity increases significantly due to the magnetic fluid lubricant. It is seen from Figure 6 that the effect of ψ with respect to μ^* is negligible up to the value of porosity 0.001 as far as the variation of the load carrying capacity is concerned.

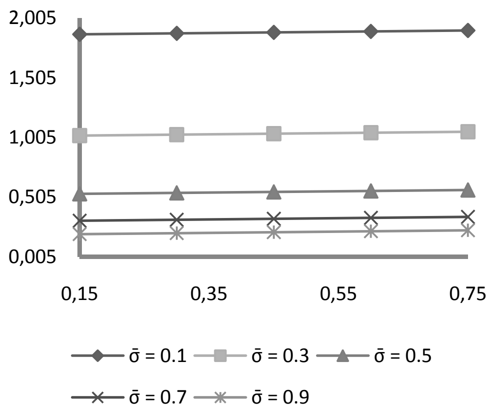

Figure 2. Variation of Load carrying capacity with respect to μ^* and $\bar{\sigma}$

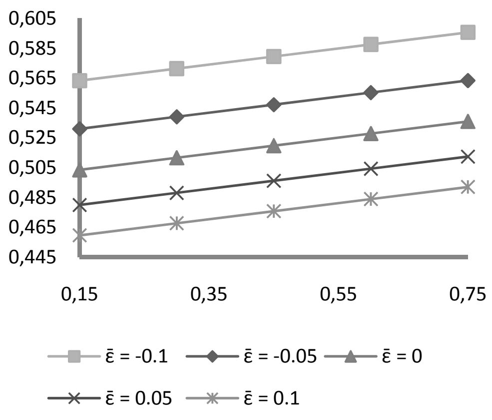

Figure 3. Variation of Load carrying capacity with respect to μ^* and $\bar{\varepsilon}$

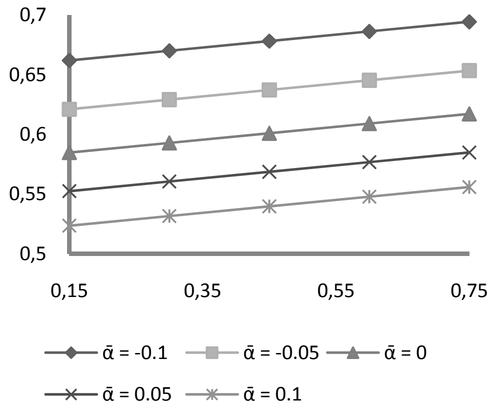

Figure 4. Variation of Load carrying capacity with respect to μ^* and $\bar{\alpha}$

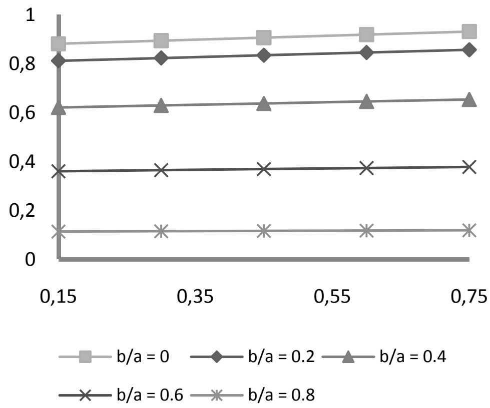

Figure 5. Variation of Load carrying capacity with respect to μ^* and b/a

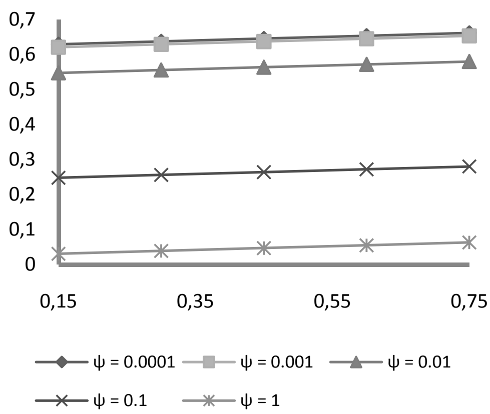

Figure 6. Variation of Load carrying capacity with respect to μ^* and ψ

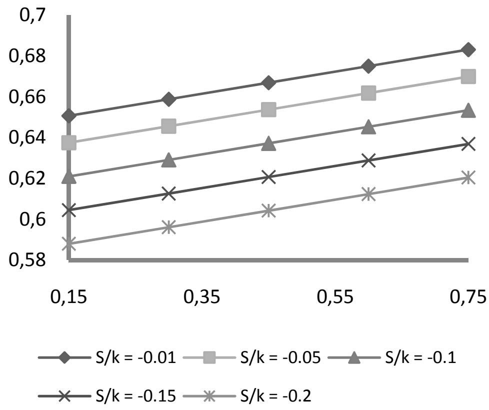

Figure 7. Variation of Load carrying capacity with respect to μ^* and s/κ

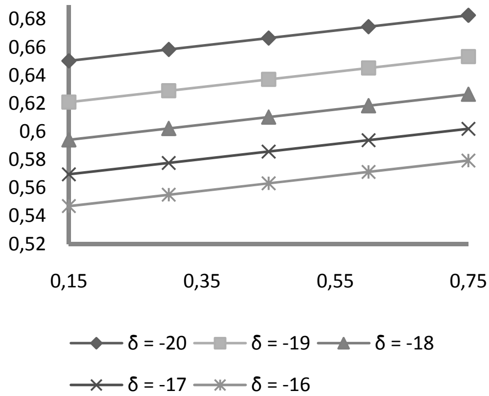

Figure 8. Variation of Load carrying capacity with respect to μ^* and $\bar{\delta}$

Figures 9-14, describe the effect of standard deviation $\bar{\sigma}$ associated with roughness on the distribution of load carrying capacity. It is clearly observed that the standard deviation adversely affects the bearing system in the sense that it decreases the load carrying capacity significantly. However, for higher values of standard deviation the effect of skewness and deformation is negligible.

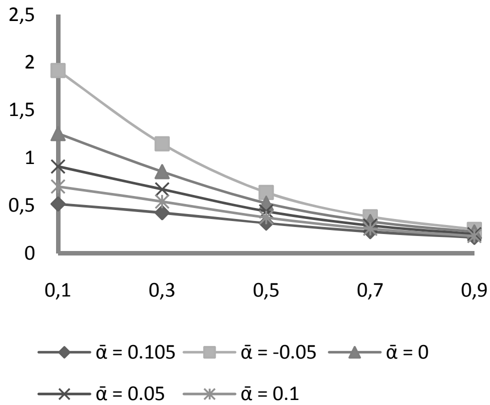

Figure 9. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\alpha}$

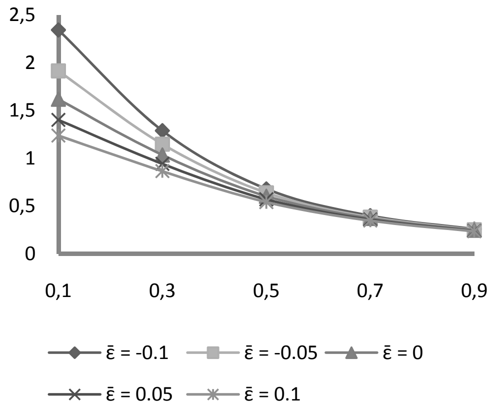

Figure 10. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\varepsilon}$

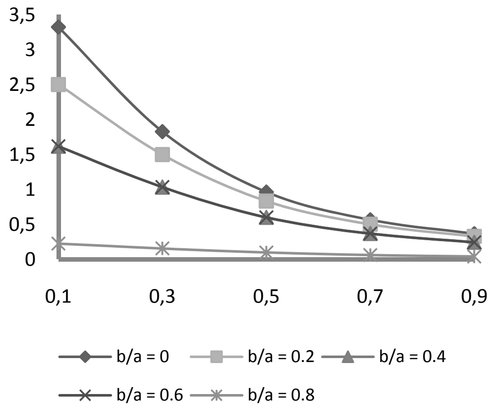

Figure 11. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and b/a

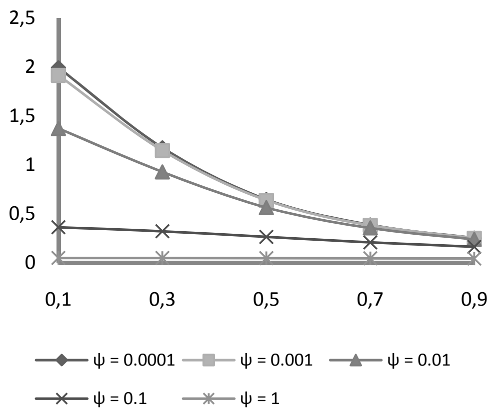

Figure 12. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and ψ

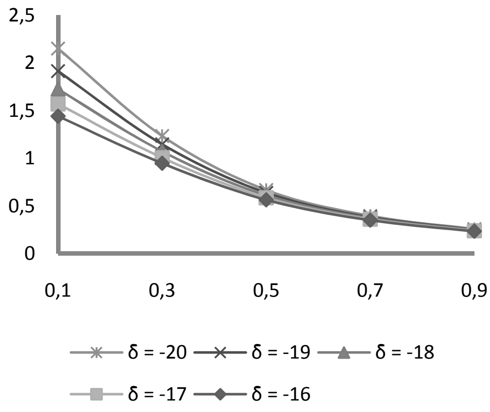

Figure 13. Variation of Load carrying capacity with respect to $\overline{\sigma}$ and $\overline{\delta}$

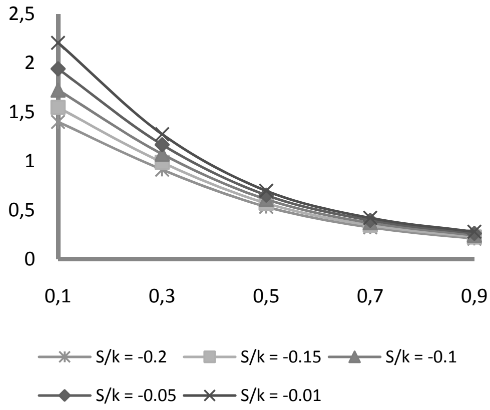

Figure 14. Variation of Load carrying capacity with respect to $\overline{\sigma}$ and s/κ

In Figures 15-20, one can observe the profile for the variation of load carrying capacity with respect to the variance$\overline{\alpha}$. It is noticed that the variance (+ ve) decreases the load carrying capacity while the load carrying capacity increases due to variance (-ve). Further, it is revealed that for higher values of $\overline{\alpha}$ the effect of $\overline{\sigma}$ is negligible while the lower values of $\overline{\sigma}$ induces a not so significant effect.

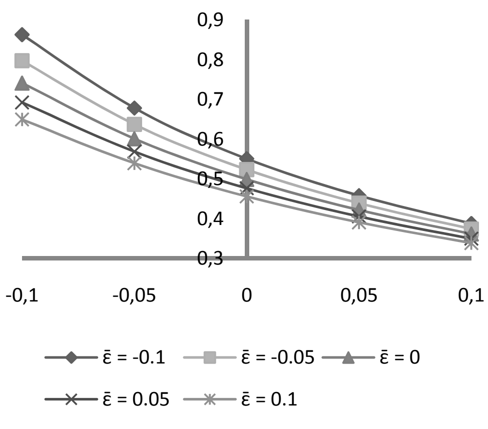

Figure 15. Variation of Load carrying capacity with respect to $\overline{\alpha}$ and $\overline{\varepsilon}$

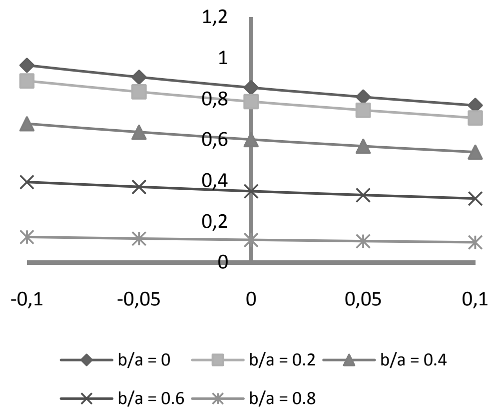

Figure 16. Variation of Load carrying capacity with respect to $\overline{\alpha}$ and b/a

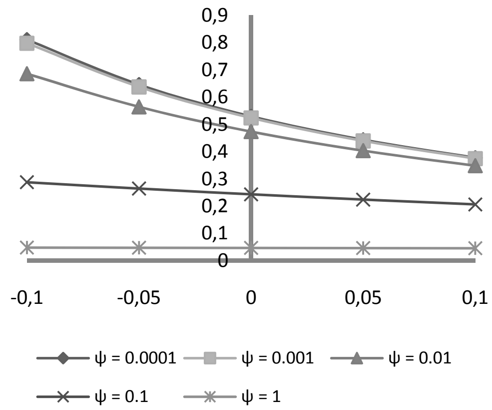

Figure 17. Variation of Load carrying capacity with respect to $\overline{\alpha}$ and ψ

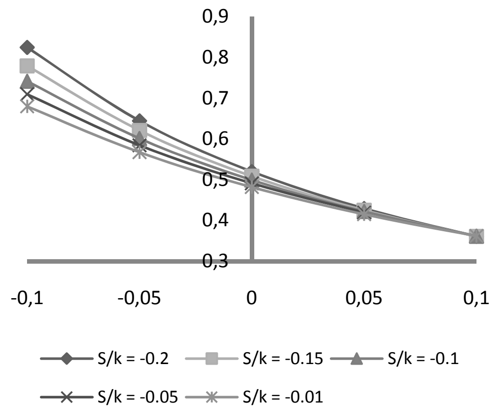

Figure 18. Variation of Load carrying capacity with respect to $\overline{\alpha}$ and s/κ

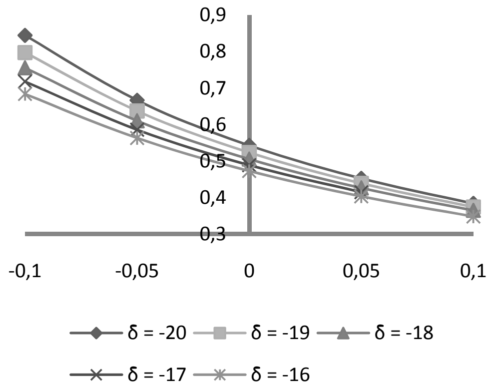

Figure 19. Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $\bar{\delta}$

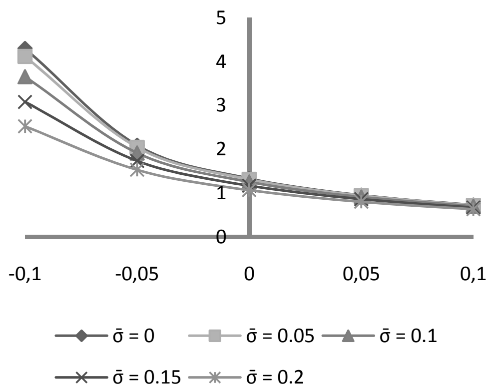

Figure 20. Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $\bar{\sigma}$

The effect of skewness on the distribution of load carrying capacity is presented in Figures 21-26. It is seen that the skewness follows the trends of $\bar{\alpha}$. These make it clear that the combined effect of variance (- ve) and negatively skewed roughness is considerably positive because the increase in load carrying capacity due to variance (- ve) gets further increased owing to negatively skewed roughness.

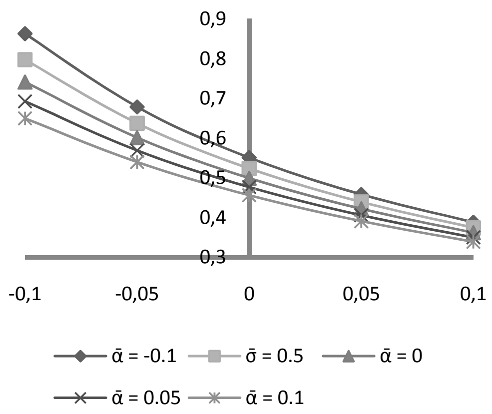

Figure 21. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\bar{\alpha}$

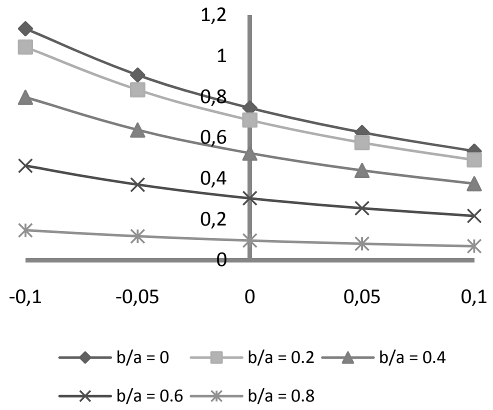

Figure 22. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and b/a

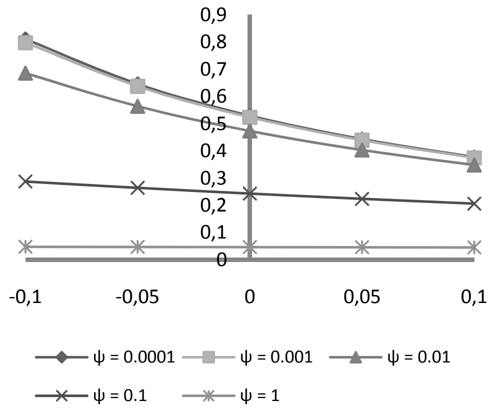

Figure 23. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and ψ

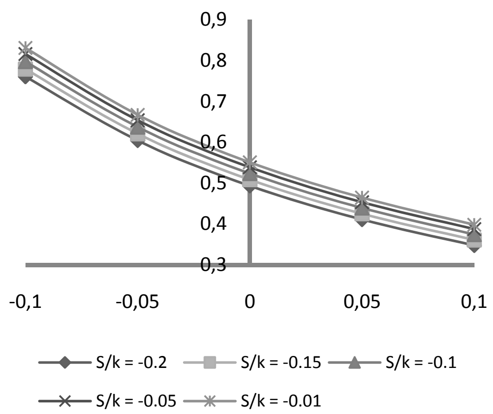

Figure 24. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and s/κ

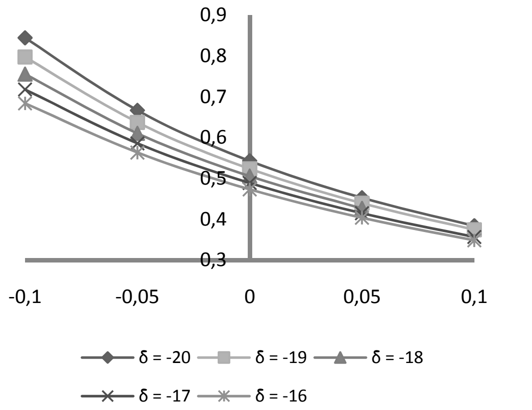

Figure 25. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\bar{\delta}$

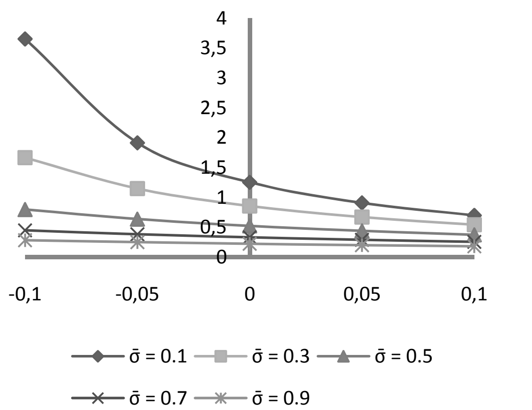

Figure 26. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\bar{\sigma}$

The effect of pocket radius on the distribution of load carrying capacity is depicted in Figures 27-29. It is clearly observed that the load carrying capacity decreases substantially as the pocket size increases. However, the effect of rotational inertia s/κ and deformation $\bar{\delta}$ is negligible for higher values of pocket radius (Figure 29). It is found that the load carrying capacity decreases substantially due to porosity and this decreasing nature becomes all the more significant when higher values of pocket radius is involved.

Lastly, Figures 30 and 31 deal with the effect of the rotational inertia with respect to magnetization and deformation respectively. It is noticed that the increased load carrying capacity due to the rotational inertia gets further increased due to the decreasing values of the deformation parameter.

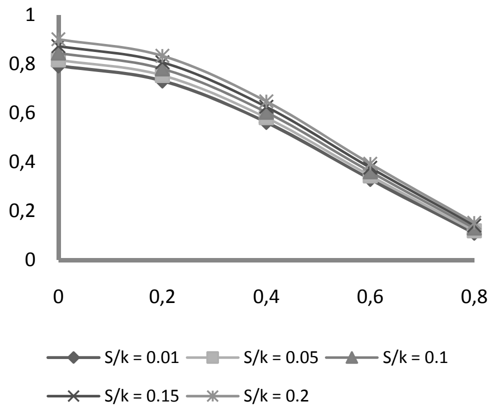

Figure 27. Variation of Load carrying capacity with respect to b/a and s/κ

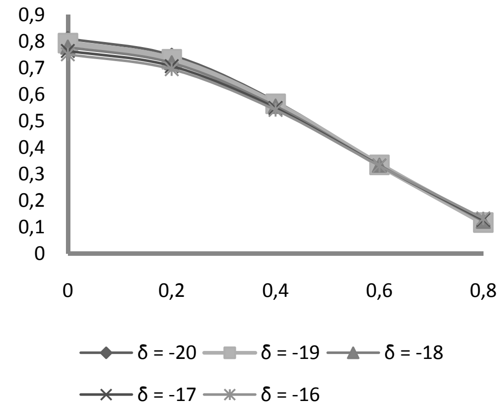

Figure 28. Variation of Load carrying capacity with respect to b/a and $\bar{\delta}$

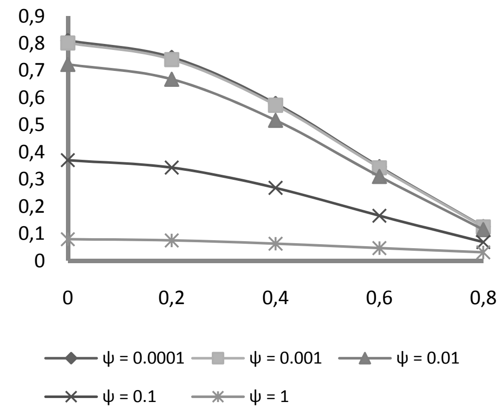

Figure 29. Variation of Load carrying capacity with respect to b/a and ψ

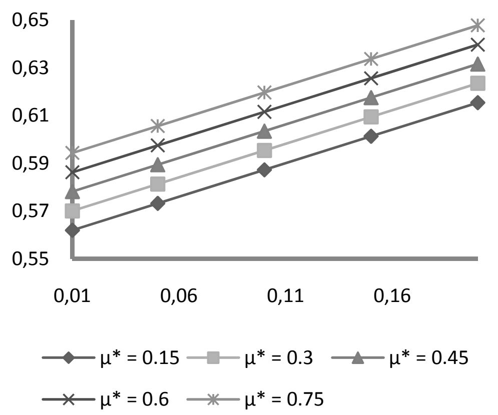

Figure 30. Variation of Load carrying capacity with respect to s/κ and μ^*

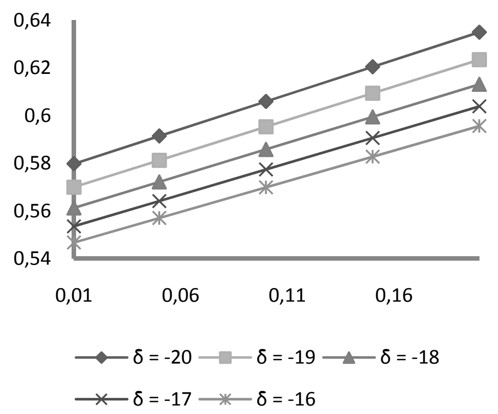

Figure 31. Variation of Load carrying capacity with respect to s/κ and $\bar{\delta}$

Some of these figures reveal that the negative effect induced by porosity, standard deviation and deformation can be reduced by the positive effect of the magnetic fluid lubricant in the case of negatively skewed roughness by suitably choosing the rotational inertia and the pocket radius. This reduction is more evident when variance (- ve) and lower values of deformation occur.

4. CONCLUSION

This article reveals that the bearing can support the load even when there is no flow. The increase in the pocket radius resulting from wear or design consideration reduces a load carrying capacity of the bearing as a result of which for a constant applied load the bearing surfaces approach each other more rapidly. In addition, this investigation suggests that while designing the bearing system, roughness must be accorded due consideration.

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