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RESEARCH

Shliomis Model Based Ferrofluid Lubrication of a Rough Porous Convex Pad Slider Bearing

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A B S T R A C T

An attempt has been made to analyze the performance characteristics of a Shliomis model based ferrofluid lubrication of a rough porous convex pad slider bearing. Regarding roughness, the stochastic method adopted by Christensen and Tonder finds the application here in statistical averaging of the associated Reynolds equation. The graphical representation suggests that the adverse effect of surface roughness can be reduced to certain extent by the positive effect of Shliomis model based ferrofluid lubrication. Further, for this type of bearing system, this model remains more effective as compared to Neuringer-Rosensweig model.

1. INTRODUCTION

Lubrication in bearings is well known in industrial applications and their main advantages compared with rolling and friction bearings are their role in reduction in friction and resorting in very high precision. Various applications and the investigations of the reheological properties of magnetic fluids gained noticeable importance during the last few years. Nowadays, the flow and the application possibilities of suspensions of magnetic nano particles are an extremely lively research field.

Porous sliders are important in fluid cushioned moving pads. Applications of porous bearings in mounting horsepower motors include vacuum cleaners, water pumps, record players, tape recorders and generators. The analyses of porous slider bearings have been based upon the Darcy's model, where Darcy's equations were applied to guide fluid motion through the porous medium [1-6].

Conventional fluid based lubricants were used in above studies. The application of magnetic fluid as a lubricant was studied by a number of authors [7-12]. In all these studies it has been noticed that the performance of the bearing system could be enhanced by using a magnetic fluid as the lubricant.

Shliomis [13] developed a ferrofluid flow model, in which the effects of rotation of magnetic particles, their magnetic moments and the volume concentration were included. Kumar et al. [14] studied the effect of a ferrofluid squeeze film for spherical and conical bearings with a constant external magnetic field applied in the direction transverse to that of fluid flow. Singh and Gupta [15] theoretically investigated the effect of ferrofluid on the dynamic characteristics of curved slider bearings using Shliomis model. On the ground of the ferrohydrodynamic model proposed by Shliomis [13], Lin [16] discussed the influence of fluid inertia forces on the ferrofluid squeeze film between a sphere and a plate in the presence of external magnetic fields. It was established from the above studies that the volume concentration and the intensity of magnetic field provided an increase in the load carrying capacity and the time of approach.

Surface roughness evaluation is very essential for many fundamental problems such as friction, load carrying capacity, contact deformation, heat and electric current conditions, tightness of contact joints and positional accuracy. Tzeng and Seibel [17] realized the random nature of roughness orientation and developed a stochastic method to describe the surface roughness. Later on, this was modified by Christensen and Tonder [18-20] to propose a more general method for analyzing the effect of both the roughness patterns (transverse as well as longitudinal). This method was adopted by many investigators [21-30] analyzed the effect of Shliomis model based ferrofluid lubrication on the squeeze film between curved rough annular plates with comparison between two different porous structures. Kozeny- Carman's formulation and Irmay's model were treated for porous structures. It was noticed that the effect of morphology parameter and volume concentration parameter increased the load carrying capacity of the bearing system. Patel and Deheri [31] dealt with the effect of different porous structures on the performance of a Shliomis model based magnetic squeeze film in rotating rough porous curved circular plates. It was seen that the adverse effect of transverse roughness could be compensated by the positive effect of magnetization in the case of negatively skewed roughness, suitably choosing the rotation ratio and the curvature parameters. Patel and Deheri [32] presented an analytical solution for the performance characteristics of a magnetic fluid based double layered porous rough slider bearing. It was observed that the increased load carrying capacity owing to double layered got enhanced due to the magnetic fluid lubricant and this went a long way in reducing the adverse effect of roughness in the case of Kozeny-Carman model.

In the existing literature Neuringer-Rosensweig model based ferrofluid lubrication of a porous convex pad slider bearing has been discussed.

Also, it is well known that the roughness affects the bearing system significantly. One gets the information from the literature regarding ferrofluid flow that, Shliomis model registers an improved performance as compared to Neuringer-Rosensweig model. Thus, it was thought proper to analyze the performance characteristics of a Shliomis model based ferrofluid lubrication of a rough porous convex pad slider bearing.

2. ANALYSIS

The configuration of plate slider bearing with squeeze velocity $\dot{h} = dh/dt$ is displayed in Fig. 1. The lower surface is a slider of length A and moving with uniform velocity U in the x direction. Also, the slider is having width B in the y direction with $A \ll B$. Moreover, h_2 and h_1 are maximum and minimum film thicknesses respectively.

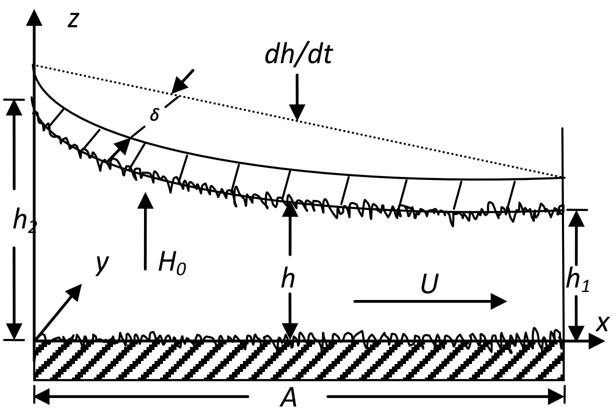

Fig. 1. Configuration of the bearing system.

The bearing surfaces are considered to be transversely rough. In line with the discussions of Christensen and Tonder [18-20], the thickness h(x) of the lubricant film is taken in the form of:

$$\mathrm{h}(\mathrm{x}) = \bar{\mathrm{h}}(x) + h_s \quad (1)$$

where $\bar{\mathrm{h}}(x)$ denotes the mean film thickness and h_s is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. h_s is considered to be stochastic in nature and governed by the probability density function:

$$f(h_s) = \begin{cases} \dfrac{35}{32c}\left(1 - \dfrac{h_2^2}{c^2}\right)^3, -c \le h_s \le c \\ 0, elsewhere \end{cases}$$

wherein c is the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε which is the measure of symmetry of the random variable h_s are defined and discussed in detail in Christensen and Tonder [18-20].

The basic equations governing Shliomis model based ferrofluid lubrication are discussed [13,30] by:

$$-\nabla p + \eta \nabla^2 \bar{q} + \mu_0 (\bar{M}\nabla)\bar{H} + \frac{1}{2\tau_s} \nabla \times (\bar{S} - I\bar{\Omega}) = 0 \quad (2)$$

$$\bar{\Omega} = \frac{1}{2} \nabla \times \bar{q} \quad (3)$$

$$\bar{S} = I\bar{\Omega} + \mu_0 \tau_s (\bar{M} \times \bar{H}) \quad (4)$$

$$\bar{M} = M_0 \frac{\bar{H}}{H} + \frac{\tau_B}{I} (\bar{S} \times \bar{M}) \quad (5)$$

$$\nabla \times \bar{H} = 0 \quad (6)$$

$$\nabla (\bar{H} + \bar{M}) = 0 \quad (7)$$

where p represents the pressure, η is the viscosity of the suspension, μ_0 denotes the permeability of free space, $\bar{H}$ is the applied magnetic field, $\bar{M}$ is the magnetization vector, $\bar{q}$ denotes the fluid velocity, $\bar{S}$ represents internal angular momentum, I is the sum of moments of inertia of the particle per unit volume, τ_B is the Brownian relaxation time, τ_s is the magnetic moments relaxation time and M_0 denotes the equilibrium magnetization.

Combining all above equations, one can get:

$$-\nabla p + \eta \nabla^2 \bar{q} + \mu_0 (\bar{M}\nabla)\bar{H} + \frac{1}{2}\mu_0 \nabla \times (\bar{M} \times \bar{H}) = 0 \quad (8)$$

$$\bar{M} = M_0 \frac{\bar{H}}{H} + \tau_B \bar{\Omega} \times \bar{M} - \frac{\mu_0 \tau_B \tau_s}{I} \bar{M}(\bar{M} \times \bar{H}) \quad (9)$$

For the strong magnetic field Langevin's parameter $\xi > 1$, the above equation takes the form,

$$\bar{M} = \frac{M_0}{H}\left[\bar{H} + \bar{\tau}(\bar{\Omega} \times \bar{H})\right] \quad (10)$$

with

$$\bar{\tau} = \frac{6\eta\phi}{nk_B T(1 + \xi \coth \xi)} \quad (11)$$

where

$$M_0 = n\mu\left(\coth \xi - \frac{1}{\xi}\right), H = \frac{k_B T \xi}{\mu_0 \mu}, \quad (12)$$

for a suspension of spherical particles:

$$\frac{I}{\tau_s} = 6\eta\phi \text{ and } \tau_B = \frac{3\eta V}{k_B T}, \quad (13)$$

$\phi = nV$ is the volume concentration of the particles, k_B is the Boltzmann constant, n denotes the number of particles per unit volume, V is the volume of the particle, T represents the temperature and μ is the magnetic moment of a particle.

Under the uniform magnetic field $\bar{H} = (0, 0, H_0)$, equations (8-10) yield

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta(1+\tau)} \frac{dp}{dx} \quad (14)$$

where

$$\tau = \frac{3}{2}\phi \frac{\xi - \tanh \xi}{\xi + \tanh \xi} \quad (15)$$

Solving equation (14) under the no slip boundary conditions:

$u = 0$ at $z = h$ and $u = U$ at $z = 0$,

one can find

$$u = \frac{1}{\eta(1+\tau)}\left(\frac{z^2}{2} - \frac{h}{2} z\right)\frac{\partial p}{\partial x} + U\left(1 - \frac{z}{h}\right) \quad (16)$$

Substituting u in the integral form of the continuity equation for the film region:

$$\frac{\partial}{\partial x}\int_0^h u dz + w_h - w_0 = 0 \quad (17)$$

yields

$$\frac{d}{dx}\left[-\frac{h^3}{12\eta(1+\tau)}\frac{dp}{dx} + \frac{Uh}{2}\right] = \dot{h} \quad (18)$$

considering

$w_h = -\dot{h}$ and $w_0 = 0$

as the lower plate is impermeable.

If η_0 represents the viscosity of the main liquid, the viscosity of the suspension is given by the Einstein formula [13]:

$$\eta = \eta_0\left(1+\frac{5}{2}\phi\right) \qquad (19)$$

Equations (18) and (19) lead to:

$$\frac{d}{dx}\left[-\frac{h^3}{12\eta_0\left(1+\frac{5}{2}\phi\right)(1+\tau)}\frac{dp}{dx}+\frac{Uh}{2}\right]=\dot{h} \quad (20)$$

The usual assumptions of hydromagnetic lubrication [33,34,25] are made. Now, the often used well-known stochastic averaging method of Christensen and Tonder [18-20] transforms equation [20] to the modified Reynolds equation governing the pressure distribution:

$$\frac{d}{dx}\left[-\frac{g(h)}{12\eta_0\left(1+\frac{5}{2}\phi\right)(1+\tau)}\frac{dp}{dx}+\frac{Ug(h)^{\frac{1}{3}}}{2}\right]=\dot{h} \quad (21)$$

where

$$g(h)=h^3+3h^2\alpha+3\left(\alpha^2+\sigma^2\right)h+3\sigma^2\alpha+\alpha^3+\varepsilon+12\varphi H$$

Introducing the nondimensional quantities,

$$X=\frac{x}{A},\bar{h}=\frac{h}{h_1}=4\bar{\delta}X^2-\left(a-1+4\bar{\delta}\right)X+a,$$

$$\bar{\delta}=\frac{\delta}{h_1},P=\frac{h_1^2 p}{U\eta_0 A},D=\frac{A\dot{h}}{Uh_1},\bar{\sigma}=\frac{\sigma}{h_1},\bar{\alpha}=\frac{\alpha}{h_1},$$

$$\bar{\varepsilon}=\frac{\varepsilon}{h_1^3},\psi=\frac{\varphi H}{h_1^3} \qquad (22)$$

and solving above equation subject to the boundary conditions:

$$P(0)=P(1)=0 \qquad (23)$$

One arrives at the dimensionless pressure as:

$$P=\int_0^X\frac{F}{G}dX-D\int_0^X\frac{X}{G}dX-Q\int_0^X\frac{1}{G}dX \qquad (24)$$

where

$$G=\frac{g(\bar{h})}{E},E=12\left(1+\frac{5}{2}\phi\right)(1+\tau),F=\frac{g(\bar{h})^{\frac{1}{3}}}{2}$$

$$Q=\frac{\int_0^1\frac{F}{G}dX-D\int_0^1\frac{X}{G}dX}{\int_0^1\frac{1}{G}dX}$$

$$g(\bar{h})=\bar{h}^3+3\bar{h}^2\bar{\alpha}+3\left(\bar{\alpha}^2+\bar{\sigma}^2\right)\bar{h}+3\bar{\sigma}^2\bar{\alpha}+\bar{\alpha}^3+\bar{\varepsilon}+12\psi$$

The non dimensional load carrying capacity then can be obtained as:

$$W=\frac{h_1^2 w}{BUA^2\eta_0}=\int_0^1\frac{F}{G}(1-X)dX$$

$$-D\int_0^1\frac{X}{G}(1-X)dX-Q\int_0^1\frac{1}{G}(1-X)dX \qquad (25)$$

3. RESULTS AND DISCUSSION

As can be seen, the magnetization increases the viscosity of the lubricants which causes increased pressure and hence load carrying capacity. Further, for smooth bearing system, this discussion results in the investigation presented in Bhat (2003).

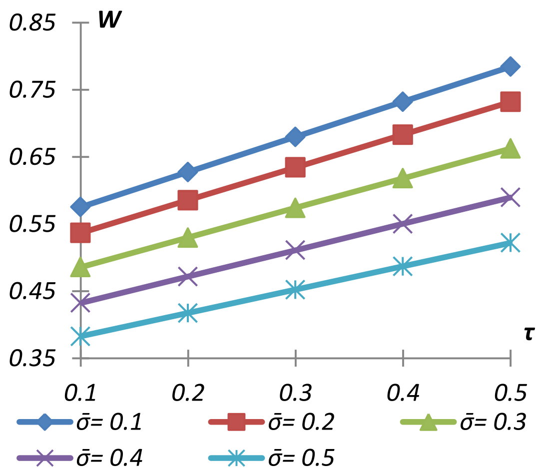

Fig. 2. Variation of Load carrying capacity with respect to τ and $\bar{\sigma}$.

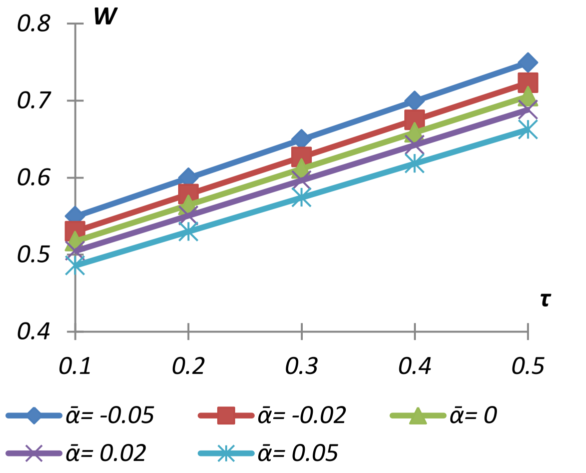

Fig. 3. Variation of Load carrying capacity with respect to τ and $\overline{\alpha}$.

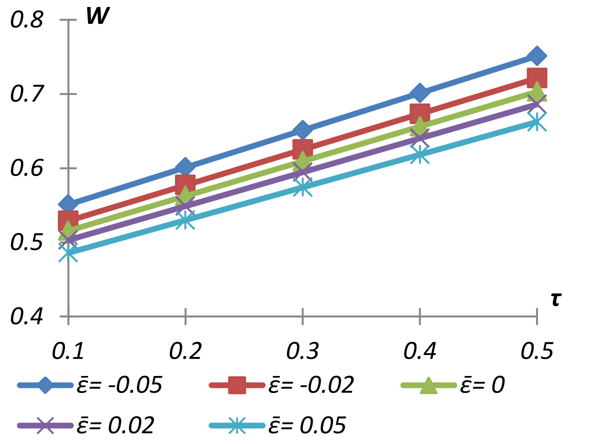

Fig. 4. Variation of Load carrying capacity with respect to τ and $\overline{\varepsilon}$.

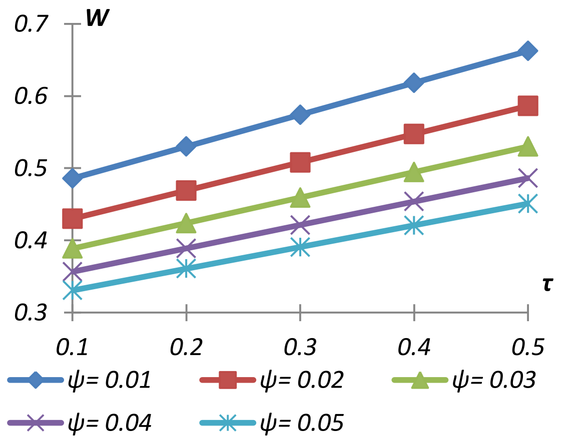

Fig. 5. Variation of Load carrying capacity with respect to τ and ψ.

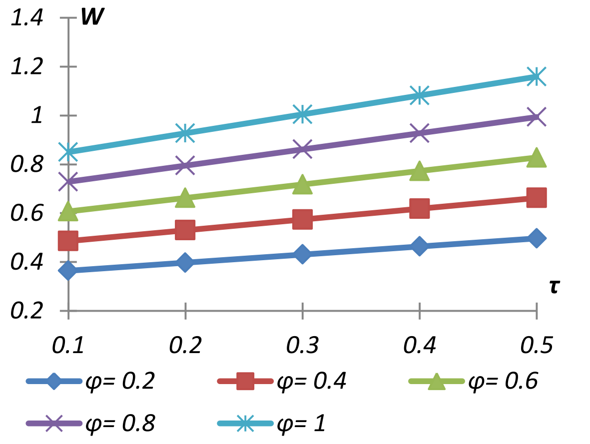

Fig. 6. Variation of Load carrying capacity with respect to τ and ϕ.

The variation of load carrying capacity with respect to magnetization parameter displayed in Figs. 2-6 indicates that the magnetization increases the load carrying capacity significantly. This increase in load carrying capacity due to the magnetization parameter is not surprising because a close look at equation (25) reveals that the expression is linear with respect to τ hence, an increase in τ would lead to increased load carrying capacity.

The effect of standard deviation depicted in Figs. 7-10 suggests that the standard deviation has a considerable adverse effect on the performance of the bearing system in the sense that it decreases the load carrying capacity considerably. The effect of higher values of porosity on load carrying capacity is negligible with respect to the standard deviation, as can be noticed. This reduction in load carrying capacity is due to the fact that the roughness retards the motion of the lubricant affecting the system adversely.

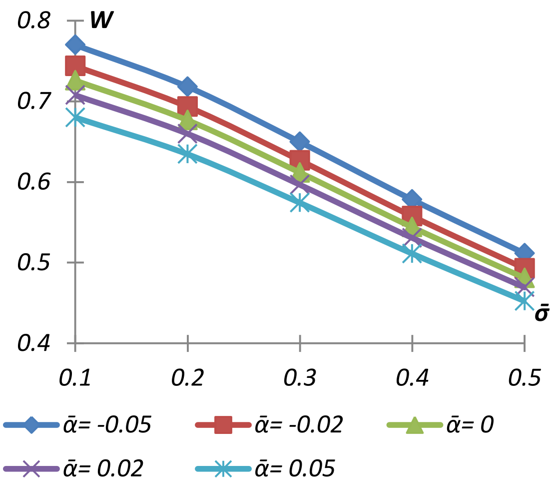

Fig. 7. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\alpha}$.

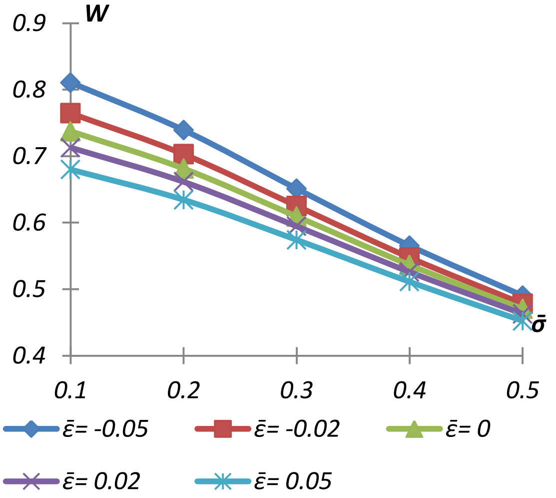

Fig. 8. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\varepsilon}$.

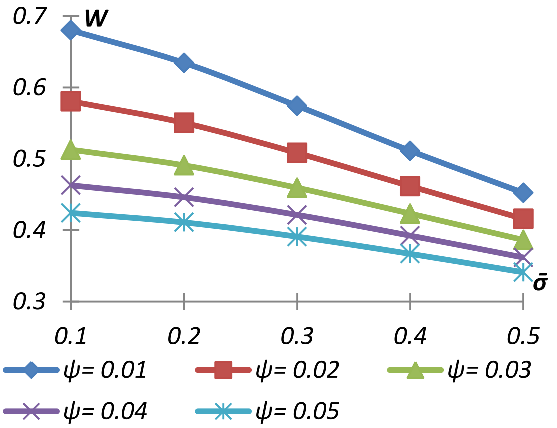

Fig. 9. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and ψ.

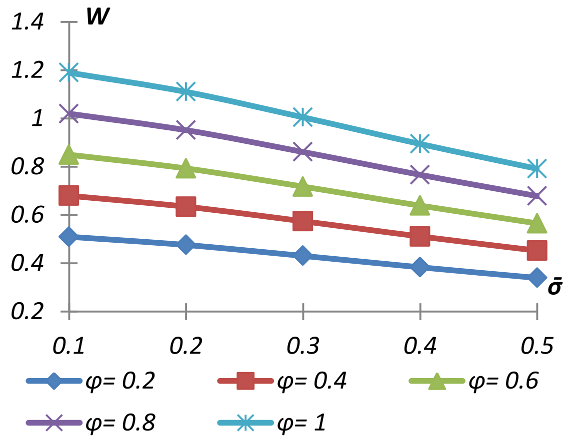

Fig. 10. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and ϕ.

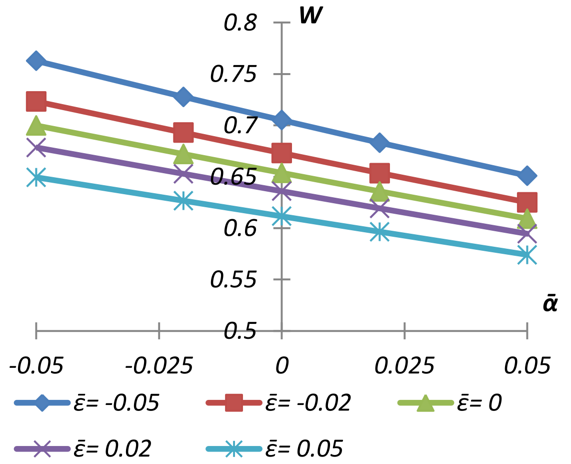

Fig. 11. Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $\bar{\varepsilon}$.

The effect of the variance on the distribution of load carrying capacity is presented in Figs. 11-13. It is clearly observed that the variance (+ve) decreases the load carrying capacity while the variance (-ve) increases the load carrying capacity. Further, it can be seen that the adverse effect of variance (+ve) can be minimized by the magnetization effect resorting to a suitable value of volume concentration parameter.

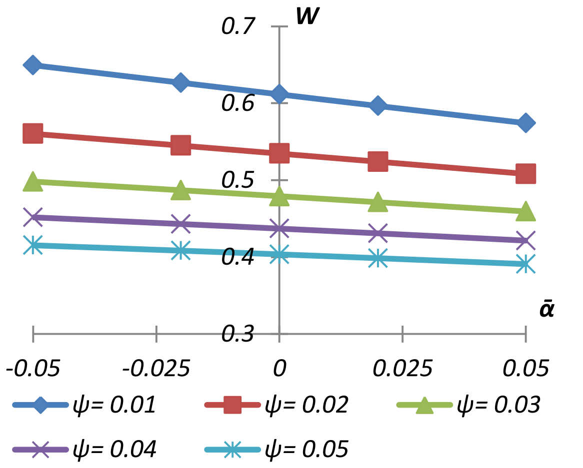

Fig. 12. Variation of Load carrying capacity with respect to $\bar{\alpha}$ and ψ.

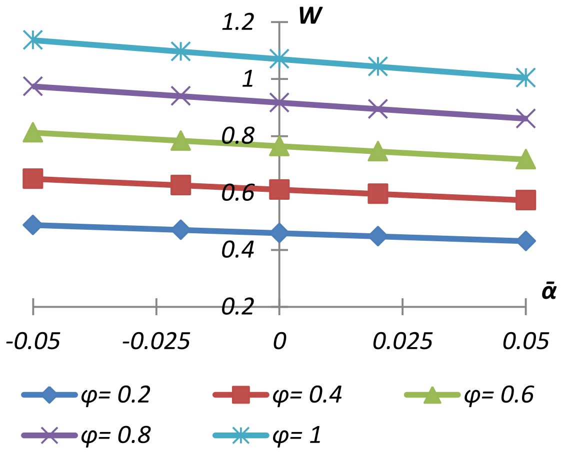

Fig. 13. Variation of Load carrying capacity with respect to $\bar{\alpha}$ and ϕ.

The variation of load carrying capacity with respect to volume concentration parameter presented in Figs. 14-16 indicates that the volume concentration parameter increases the load carrying capacity significantly. It is also found from some figures that the positively skewed roughness causes decreased load while the load carrying capacity gets increased due to negatively skewed roughness. Thus, the skewness follows the trends of variance so far as load carrying capacity is concerned. Therefore, the positive effect of variance (-ve) gets a further boost in the case of negatively skewed roughness.

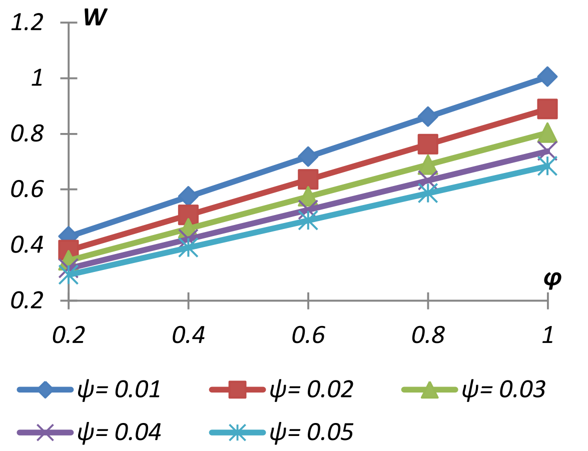

Fig. 14. Variation of Load carrying capacity with respect to ϕ and ψ.

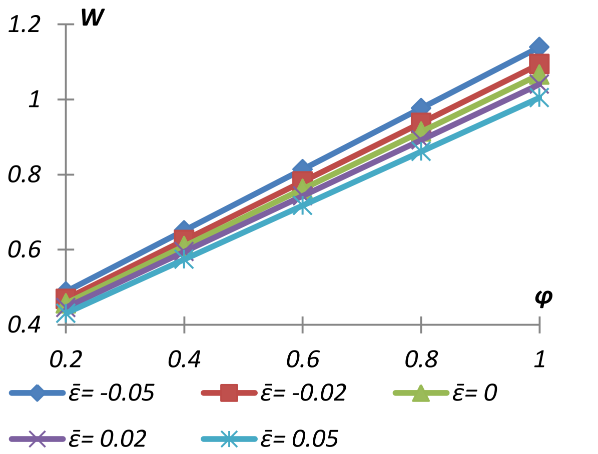

Fig. 15. Variation of Load carrying capacity with respect to ϕ and $\bar{\varepsilon}$.

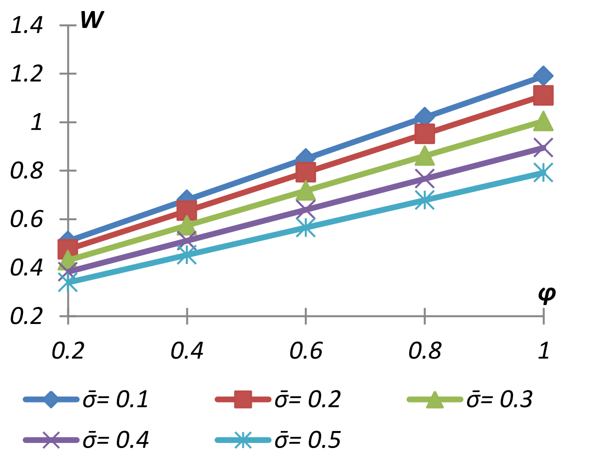

Fig. 16. Variation of Load carrying capacity with respect to ϕ and $\bar{\sigma}$.

This may be due to the fact that the motion of the lubrication gets opposed by roughness, resulting in lowering of pressure and hence the load.

As usually, observed the load carrying capacity gets decreased due to porosity, as some amount of lubricant enters into the pores.

This study offers the suggestions that the combined positive effect of variance (-ve) and negatively skewed roughness may be channelized for improving the bearing performance.

4. CONCLUSION

From this investigation it is clear that the transverse surface roughness affects the bearing performance characteristics adversely. However, this article also provides some measures in overcoming this adverse effect by the positive impact of magnetization in the case of negatively skewed roughness. Equally crucial is the role of variance (-ve) when higher values of porosity is involved. Therefore, the roughness aspect is required to be considered while designing the bearing system. It may be desirable to evaluate exclusively the contribution of the volume concentration parameter, for enhancing the bearing performance. The current study also points to the extent the magnetization can go in lowering the adverse effect of roughness for lower values of porosity. A noticeable observation is that this type of bearing system supports a good amount of load even in the absence of flow unlike the case of a conventional lubricant based bearing system. A simple calculation indicates that at least there is 20-25 % increase in the load carrying capacity in spite of the adverse effect of roughness.

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