

# The Optimization of Microgeometric Parameters of Hydrodynamic Heavy Loaded Tribounits of a Forced Internal Combustion Engine

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## ABSTRACT

The article presents a technique for solving the problem of multicriteria optimization of microgeometric parameters. The calculation methods are based on the mass conservation algorithm of lubricant layer. The description of the developed software package for solving the problem of determining optimal microgeometric parameters of a journal bearing is given. The calculations of the optimal microgeometric parameters performed for radial journal bearing of a forced internal combustion tractor engine, and the friction losses were estimated before and after solving the optimization problem.

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## 1. INTRODUCTION

The increasing fuel costs and the increasing restrictions on the environmental legislation demand the development of more efficient power plants, in particular, the internal combustion engines (ICE). In terms of energy losses of vehicles, the heavy-duty vehicles rank the second place after passenger cars, ahead of aviation, buses, sea and rail transport [1]. In the heavy vehicles, 33% of energy is lost to overcome the friction losses [2-3]. In terms of the friction losses in the internal combustion engine, the journal bearings of crankshaft take the second place after the tribounit "piston-cylinder". Thus, reducing frictional losses in

these key vehicles is one of the important goals in the engine design. The main ways to reduce these losses include the use of coatings that reduce the coefficient of friction, the improved surface technologies, the energy-efficient lubricants and the textured bearing surfaces.

In the works [4-5] it was shown that the textured surface, in comparison with the non-textured one, expanded the range of the hydrodynamic friction, and also significantly reduced the friction force in hydrodynamic units of the internal combustion engine.

The studies carried out in [6-9] indicate an increase in wear resistance, a decrease in

likelihood of jamming and the preservation integrity of contacting surfaces due to the trapping of abrasive particles trapped in the lubricating layer of tribounits by micro dimples.

Despite the fact that a large number of studies are focused on the power losses in journal bearings of rotary machines and stationary loaded tribounits [10-15], there are very few works devoted to the friction power losses and the increasing bearing capacity of heavily loaded hydrodynamic tribounits of forced ICE using the methods for optimizing the microgeometry of the friction surfaces.

This work solves the problem of optimizing the microgeometric parameters of the friction surfaces of heavy loaded hydrodynamic tribounits to increase their bearing capacity and to reduce the tribological losses in the ICE.

## 2. METHODOLOGY OF HYDRODYNAMIC CALCULATION OF TRIBOCONNECTION

The lubricating layer of a radial bearing, taking into account macrogeometric deviations, is shown on the example of a connecting rod bearing (Fig. 1). It is limited by the surfaces of bearing (liner)  $S_1$  and pin (shaft)  $S_2$ . The designations shown in the figure are described in the work [16].

To determine the field of hydrodynamic pressures, the law of conservation fluid mass at the boundaries rupture and the recovery of the lubricating layer was used.

The mass conservation law is implemented by integrating the modified Elrod equation:

$$\frac{\partial}{\partial \varphi} \left[ \frac{\bar{h}^3}{12 \bar{\mu}_s} \frac{\partial}{\partial \varphi} (g\Phi) \right] + \frac{1}{a^2} \frac{\partial}{\partial z} \left[ \frac{\bar{h}^3}{12 \bar{\mu}_s} \frac{\partial}{\partial z} (g\Phi) \right] = \frac{\bar{\omega}_{21}}{2} \frac{\partial}{\partial \varphi} \{ \bar{h} [1 + (1-g)\Phi] \} + \frac{\bar{\omega}_{21}}{2} \frac{\partial}{\partial z} \{ \bar{h} [1 + (1-g)\Phi] \} + \frac{\partial}{\partial \tau} \{ \bar{h} [1 + (1-g)\Phi] \}, \quad (1)$$

Where  $\bar{h} = h/h_0$ ;  $\bar{\mu}_s^* = \mu_s^*/\mu_0$ ;  $-a \leq \bar{z} \leq a$ ;  $\bar{z} = z/R$ ;

$$a = B/2R; \bar{\omega}_{21} = (\omega_2 - \omega_1)/\omega_0; \tau = \omega_0 t; \bar{\omega}_{21} = (\omega_2 - \omega_1)/(\omega_0 R);$$

$\bar{h}, \bar{\mu}_s$  - the dimensionless lubricant layer thickness and the effective lubricant viscosity;

$B, R$  - the bearing width and radius;

$\mu_E^*$  - the effective lubricant viscosity corresponding to temperature  $T_E^*$ ;

$\mu_0, h_0, \omega_0$  - the lubricant viscosity, the characteristic thickness of lubricant layer at the center position of pin and the pin rotational speed;

$\bar{\omega}_{21}, \omega_{21}$  - the dimensionless angular and translational speeds of pin;

$g$  - switching function.

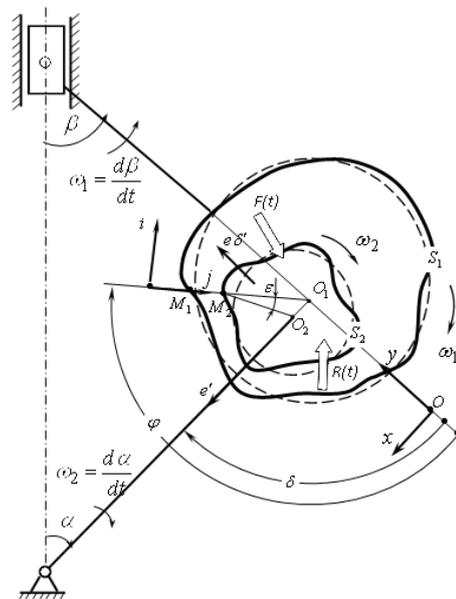


Fig. 1. The connecting rod bearing diagram.

The boundary conditions of Jacobson-Floberg-Olsen (JFO):

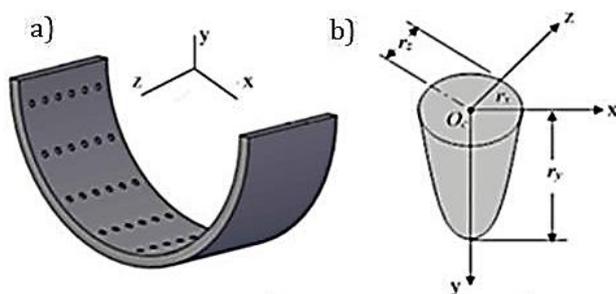
$$\bar{p}(\varphi, \bar{z} = \pm a) = 0; \bar{p}(\varphi, \bar{z}) = \bar{p}(\varphi + 2\pi, \bar{z}); \bar{p}(\varphi_p, \bar{z}) = \partial \bar{p} / \partial \varphi(\varphi_p, \bar{z}) = 0; \bar{p}(\varphi_e, \bar{z}) = 0. \quad (2)$$

Where  $\varphi_p$  - break angle;  $\varphi_e$  - recovery angle.

The main hydromechanical characteristics (HMC) of the heavy loaded tribounits are: the average values of power losses  $N^*$ , the minimum thickness of lubricating layer  $h_{min}^*$ , the maximum hydrodynamic pressure  $p_{max}^*$ , the lubricant consumption  $Q_T^*$ , the

temperature of lubricating layer  $T_E^*$ , as well as the maximum value of hydrodynamic pressure per cycle  $\sup p_{\max}$  and the lowest value of the minimum thickness of lubricating layer per cycle  $\inf h_{\min}$ .

One of the ways to reduce the mechanical losses and the oil starvation can be texturing of contacting surfaces (Fig. 2, 3), which will increase the bearing capacity of a bearing with a heavy load due to creation of many "micro wedges". In particular, surface's texturing of crankshaft bearing shells can be performed in the form of elliptical pits, and the surface of a piston skirt in the form of micro grooves of various shapes, which allows the oil to remain on the friction surface under any engine operating conditions.



**Fig. 2.** The textured bearing surface (a) with dimensions of micro dimples (b).

The thickness of lubricating layer for a radial bearing, taking into account both micro and macro geometry of surface, can be represented as:

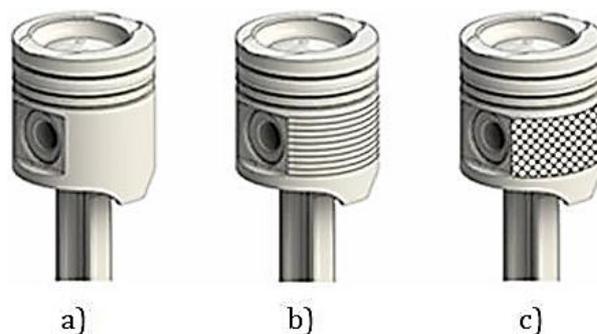
$$h(\phi, z) = h_0(\phi, z) + \Delta h(\phi, z), \quad (3)$$

where  $h_0(\phi, z)$  – bearing's macrogeometry;  $\Delta h(\phi, z)$  – surface's microgeometry. The mathematical description of geometry of micro dimples is given in the work [3].

The change in the thickness of lubricating layer corresponds to the change in the  $y$  coordinate and obeys the following condition [17]:

$$\Delta h(\phi, z) = \begin{cases} \frac{r_y}{r} \sqrt{r^2(x^2 - x_c)^2 - (z - z_c)^2}, & \text{if } \sqrt{x^2 + z^2} \leq r; \\ 0, & \text{if } \sqrt{x^2 + z^2} \geq r, \end{cases} \quad (4)$$

where  $z_c, x_c$  – the center coordinate of micro dimples;  $r$  – the radius of micro dimples.



**Fig. 3.** The types of regular microgeometry of piston skirt: a) smooth surface; b) radial micro grooves; c) cross microgeometry.

According to the condition (4), the increase in thickness of the lubricating layer  $\Delta h(\phi, z)$  corresponds to the change in the  $y$  coordinate within the boundaries of micro dimple at the point with coordinates  $(x, z)$  if the distance from the center of micro dimple to its boundaries is less than its radius, otherwise the thickness of lubricating layer will not change.

### 3. METHODOLOGY OF OPTIMIZATION

The optimization algorithm is based on numerical monitoring of space of variable parameters, and the study can be divided into three stages.

The first stage consists in preparing the test table, which reflects the dependence of results of calculated quality criteria on the varied parameters using the equation (1) and the methods for solving the equations of motion of movable bearing elements [16]. The variable parameters are the microgeometric characteristics of radial bearing:  $r$  – micro-well radius,  $r_y$  – micro dimple depth,  $\phi_b$  and  $\phi_e$  – the beginning and the end of zone of microtexturing location along bearing angle,  $\varepsilon_b$  – micro dimples location density on the friction surface. At the second stage, the experts set limits for each of the quality criteria. The third stage involves an automatic check of non-emptiness of the set of admissible values  $(r, r_y, \phi_b, \phi_e, \varepsilon_b)$ , and if the set is empty and does not contain any combination of variable parameters, the return to the second stage is made, where the experts change the assigned restrictions or an increase number of sample points. After these changes, the optimization algorithm starts again. If the set  $(r, r_y, \phi_b, \phi_e, \varepsilon_b)$  is not empty, the experts exclude the ineffective points, and the remaining set is the Pareto set  $\tilde{\Pi}$ .

For microgeometric parameters, the constraints are set based on analysis of multivariate studies of the authors [18]. As a result, the space of variable parameters for optimization is presented in the form of the points  $q$  with the coordinates  $(r, r_y, \varphi_b, \varphi_e, \varepsilon_b)$ .

The quality criterion such as characteristic of tribounit, which is associated with its quality by the monotonic dependence, i.e. other things being equal, the system will be more reliable and efficient, the more (less) the criterion. The criteria of quality of a radial journal bearing  $\Phi_1(q), \Phi_2(q), \dots, \Phi_k(q)$  are the average for the HMC cycle:  $N^*, h_{\min}^*, p_{\max}^*, Q_T^*, T_E^*$ , which form the system quality vector  $\Phi(q)$ , taking into account the values and the importance of each of the particular criteria  $\Phi_v(q)$ , where  $v$  – the number of characteristics included in the list of the optimization criteria.

In view of the above, the optimization problem is to find a point  $\tilde{q}$  such that

$$\Phi(\tilde{q}) = \underset{q \in D}{\text{opt}} \Phi(q). \quad (5)$$

Here  $D$  – the set of admissible points that determine the optimal value of quality vector.

Note that when solving the problem of optimizing of parameters of radial journal bearings, it is necessary to perform the multivariate calculations, while the computational grid reaches the values of 500 per 2000 cells, which is associated with the high costs of the computer time.

The software package "MICRO-OPTIM" (Fig. 4) developed by authors [19] and used to optimize microgeometric parameters of a radial journal bearing includes the following main programs and subroutines:

OPTIMIZATION – the program of multicriteria optimization of microgeometric parameters by the LP- search method (LP - the asymptotically best among all known uniformly distributed sequences).

MICROGEOMETRY – the subroutine of calculating hydrodynamic pressures in the lubricating layer, taking into account microgeometric parameters.

LPTAU – the subroutine of the calculating the  $i$ -th point, the uniformly distributed sequence in the  $N$ - dimensional cube of the LP – sequence.

LPP – the subroutine selection of trial variable points.

PARET – the ineffective points of elimination subroutine.

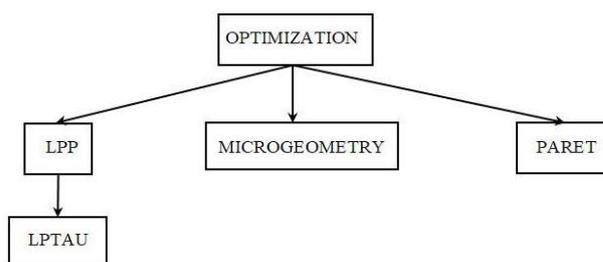


Fig. 4. The optimization software package structure.

The program is designed by authors for the tribological analysis of radial journal bearings with a textured friction surface, including the tribounit of crankshaft of a piston internal combustion engine. The initial data are: the indicator diagram of the working process in cylinder, the geometric parameters, including the microtopography of friction surfaces, the operating and operational characteristics.

#### 4. RESULTS AND DISCUSSION

The multicriteria problem of the optimization of microgeometric parameters is presented on the example of a connecting rod bearing of a crankshaft of a new tractor engine ChN 13/15.

The microtexturing parameters varied within:

$r_y = 0,000020 \text{ m} \dots 0,0005 \text{ m}$ ;  $r = 0,000776 \text{ m} \dots 0,001 \text{ m}$ ;  $\varepsilon_b = 0 \dots 70\%$ . As the initial data, the viscosity-temperature characteristic of an SAE 5W-50 engine oil was used.

In accordance with the optimization method, which includes the developed techniques described above, a range of variable microgeometric parameters was established. A set of calculations was performed from 277 options and, in accordance with the first stage of

optimization, the test table was compiled (Table 1). During the calculation, there were unsuccessful combinations of microgeometric parameters. In this case, another combination of initial data was taken and the calculation continued.

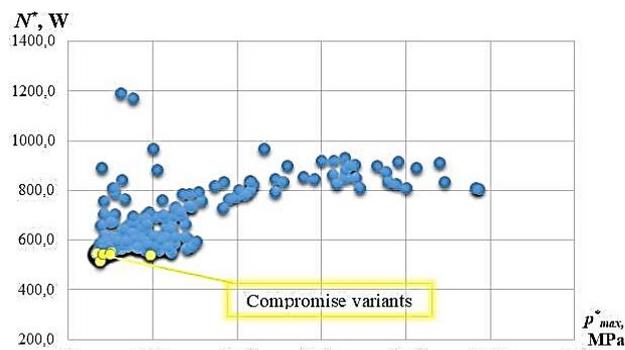
The values of criteria are based on the experience of experts and the results of preliminary calculations of analogs. During the calculating process experts assigned the following constraints to the quality criteria:

- $h_{min}^* = sup$  – greatest value ( $h_{min}^* > 2,5 \mu m$ );
- $Q_T^* = inf$  – smallest value ( $Q_T^* < 0,035 l/s$ );
- $T_E^* = inf$  – smallest value ( $T_E^* < 110 \text{ }^\circ C$ );
- $p_{max}^* = inf$  – smallest value ( $p_{max}^* < 99 \text{ MPa}$ );
- $N^* = sup$  – smallest value ( $N^* < 550 \text{ W}$ ).

Taking into account these constraints the Pareto set was determined automatically (Table 2) in the form of six compromise options for a set of variable microgeometric parameters ( $r, r_y, \varphi_b, \varphi_e, N_B, N_L$ ).  $N_B$  – the number of micro dimples along the bearing width;  $N_L$  – the number of the micro-wells around the bearing circumference. Only six options, which we selected from table 1, fulfilled the quality criteria.

**Table1.** The calculation results of connecting rod bearing of the ChN 13/15 diesel tractor engine.

№	Variable parameters						Quality criteria (HMC)		
	$\varphi_b$ , deg.	$\varphi_e$ , deg.	$r_y$ , mkm	$r$ , $\mu m$	$N_B$ , pc	$N_L$ , pc	$N^*$ , W	$h_{min}^*$ , $\mu m$	$p_{max}^*$ , MPa
1	154,7	306,2	449,4	882,2	7	28	<b>850,9</b>	<b>1,60</b>	<b>220,4</b>
2	11,5	104,0	456,5	978,5	6	15	<b>676,3</b>	<b>3,29</b>	<b>75,9</b>
3	359,4	49,4	134,0	778,5	7	10	<b>562,6</b>	<b>3,77</b>	<b>71,6</b>
4	154,3	35,1	82,4	821,3	7	48	<b>838,5</b>	<b>1,56</b>	<b>157,7</b>
5	0,5	123,7	376,3	957,4	6	21	<b>577,3</b>	<b>2,76</b>	<b>90,2</b>
...									
86	317,4	148,6	42,1	886,4	6	34	<b>608,6</b>	<b>3,14</b>	<b>85,1</b>
87	246,8	127,3	81,5	806,7	7	49	<b>604,3</b>	<b>2,68</b>	<b>85,1</b>
88	208,2	240,1	232,0	931,4	6	5	<b>591,4</b>	<b>3,56</b>	<b>73,7</b>
89	282,0	146,0	30,0	793,6	7	46	<b>584,0</b>	<b>3,29</b>	<b>73,6</b>
90	251,2	132,9	349,4	950,5	6	42	<b>607,5</b>	<b>2,12</b>	<b>120,1</b>
...									
273	243,7	63,9	102,9	980,9	6	29	<b>596,7</b>	<b>3,17</b>	<b>83,7</b>
274	235,3	248,0	310,7	834,7	7	2	<b>562,3</b>	<b>4,00</b>	<b>67,6</b>
275	192,1	301,9	20,5	917,3	6	19	<b>586,7</b>	<b>3,59</b>	<b>68,4</b>
276	348,8	11,0	123,7	961,3	6	2	<b>563,0</b>	<b>3,77</b>	<b>69,7</b>
277	187,5	193,5	77,3	830,6	7	1	<b>550,4</b>	<b>4,06</b>	<b>66,9</b>



**Fig. 5.** The Pareto - optimal solution in coordinates  $N^*$  and  $p_{max}^*$ .

Fig. 5 presented the values from Table 1 and shows the Pareto front of the compromise points, from the set of which, based on expert estimates, the optimal microgeometry is finally formed (№4, table 2). The selected option has the best characteristics, in particular, the lowest friction losses, which is one of the main research goals.

The obtained results are consistent with the conclusions of many works that investigate microtexturing. In these works, and in particular in [11], it is indicated, that friction losses are reduced to 10%. The authors of [11] point out the need to select texturing parameters. In the presented work, the developed optimization technique allows us to be done automatically. The obtained results prove this.

**Table2.** The optimization results of connecting rod bearing microgeometry of the ChN 13/15 diesel tractor engine.

No	$\varphi_b$ , deg.	$\varphi_e$ , deg.	$r_y$ , $\mu\text{m}$	$r$ , $\mu\text{m}$	$N_B$ , pc	$N_L$ , pc	$N^*$ , W	$h_{\min}^*$ , $\mu\text{m}$	$P_{\max}^*$ , MPa
1	44,4	107,0	452,3	865,9	7	12	<b>542,8</b>	<b>3,55</b>	<b>70,6</b>
2	64,0	110,4	385,7	781,5	7	9	<b>547,8</b>	<b>3,60</b>	<b>69,2</b>
3	42,8	77,2	478,4	898,3	6	6	<b>538,3</b>	<b>3,74</b>	<b>69,4</b>
4	221,0	329,1	21,7	779,3	6	19	<b>514,8</b>	<b>3,88</b>	<b>68,3</b>
5	307,7	117,3	488,9	830,1	7	33	<b>538,9</b>	<b>2,50</b>	<b>98,4</b>
6	13,1	66,7	424,3	864,6	7	10	<b>544,8</b>	<b>3,62</b>	<b>74,4</b>
Smooth bearing							<b>552.2</b>	<b>4.085</b>	<b>66,1</b>

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## 5. CONCLUSIONS

1. The method for multi-criteria optimization of microgeometric parameters using the Pareto-optimal solution has been developed. The proposed method is based on the calculation of hydrodynamic pressure field in the lubricating layer using the mass conservation algorithm.
2. The paper describes in detail the technology for solving the problem of microgeometric parameters. Based on the developed methods and technologies of optimization, the authors have created a software package. This software package is a continuation of the computer programs developed by the authors for the analysis of hydrodynamic tribo-couplings.
3. A multicriteria problem of microgeometric parameters optimization is presented on the example of a connecting rod bearing of the ChN 13/15 diesel tractor engine. The optimization results showed a decrease in friction losses by up to 7%. At the same time, other characteristics of bearing did not deteriorate. The obtained results are consistent with the conclusions of many works that investigate microtexturing.

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