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RESEARCH

Predictive Assessment of Maximum Contact Pressures in Metal-on-Polymer Hip Joint Implants by Analytical Method and FEM

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A B S T R A C T

In more than 90% of hip replacements, the acetabular cup is made of ultra-high molecular weight polyethylene (UHMWPE) and the femoral head is made of stainless steel, cobalt-chromium-molybdenum alloys, titanium or ceramic.

The primary factor in evaluating soft-on-hard endoprostheses is the maximum contact pressure as the contact strength of UHMWPE.

The level of these pressures is used as a criterion for the correct selection of the femoral head size and radial clearance in the connection during arthroplasty for a specific patient.

According to the previously developed new analytical method is proposed for estimating maximum contact pressures in thermo-diffusion nitrided (TDN) titanium GRADE 2 – UHMWPE hip joint implant. The effects of implant femoral head diameter and radial clearance on maximum contact pressures are determined for the average load. Relationships between maximum contact pressures and the above-mentioned parameters are established. A comparison is made between maximum contact pressures estimated by the proposed analytical method and numerical analysis with ANSYS (2D and 3D). The maximum contact pressures calculated using the 2D model have been found to be the lower limit, and the pressures calculated using the 3D model in ANSYS are the upper limit for analytical calculations.

Therefore, it should be expected that at thickness of cup ≈ 7...8 mm the contact stresses will not be significantly different than those obtained from the solution by the above method. The developed analytical method can be effectively used for research not only soft-on-hard endoprostheses (metal-on-polymer, ceramics-on-polymer), but also hard-on-hard endoprostheses (metal-on-metal, ceramics-on-ceramics). The results of analytical calculations were also related to the results of FEM calculations known in the literature and their convergence was established.

1. INTRODUCTION

There are more than one million hip joint replacement surgeries every year. The selection of a hip joint implant must be adjusted to the patient's weight (total joint load), implant component diameter and radial clearance. These factors have impact on maximum contact pressures and wear of the implant. The aforementioned factors determine the level of the maximum contact pressure generated in the endoprosthesis, and those, respectively, of its service life and wear.

In literature of the subjects lacks well-established solutions for contact pressure determination using effective computational methods based on classical contact mechanics. On the other hand, there are well-known solutions that use numerical methods, particularly Finite Element Method (FEM) [1-11]. In [9] an approximation relationship is given, obtained in calculations performed in the ANSYS module. It should be emphasized that quantitative results of maximum contact pressures and their distribution significantly differ depending on the numerical method used to model contact interactions in the hip joint implant. To verify obtained results, adequate analytical methods must be employed.

Unfortunately, the literature lacks effective analytical methods based on classic methods of contact mechanics that ensure a reliable assessment of the pressures. Various approximate approaches from the point of view of contact mechanics are presented in [12-17]. In [12], the Airy function was used for this purpose. Instead, in [13,14] a simplified calculation method is presented, which is based on an improved method from an old work [15]. In [16,17], using the Winkler elastic base model, an analytical solution for calculating the maximum contact pressures in an endoprosthesis with a UMHPE cup is presented.

There is no closed analytical solution for the almost conform contact of the sphere with the spherical cup.

Various types of simplifications in the methods used to estimate contact pressures in hip joint endoprostheses, both numerical and analytical, are indicated in [18]. In particular, in analytical models, these simplifications are: the ideal stiffness of contacting bodies, pressure distribution acording to Hertz, direct proportionality of pressure to displacement, etc.

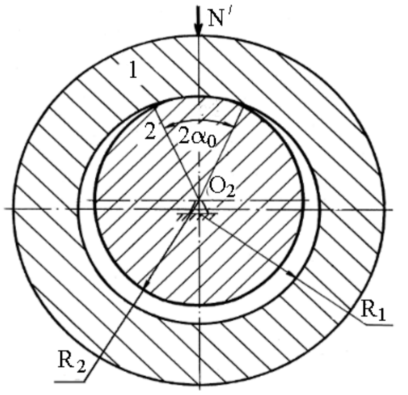
This paper presents analytical method of this type for the estimation of contact parameters in hip joint endoprostheses based on the works [19-22]. On this basis, their evaluation was carried out in the case of the metal-on-polymer prosthesis. Also using ANSYS, numerical evaluation of the model replacement 2D and 3D cylindrical joint was performed. Its purpose is to confront the results of the solution with the analytical method given below. This method was used in [23,24] to study the maximum contact pressures occurring in metal-on-UHMWPE endoprostheses.

2. MODEL OF A CONTACT PROBLEM

A hip joint implant (Fig. 1a) is a 3D cup-and-ball joint. This implant is modeled as a cylindrical joint with a fixed cup (3D system) described by a femoral head diameter of $D_2 = 2R_2$ and an effective radius. The 3D cylindrical joint is then converted into a two-dimensional (2D) system by reducing the total compressive load N acting on the implant head to the head diameter, i.e., $N' = N / D_2$ (Fig. 1b). This system is described by a small radial clearance of $\varepsilon = R_1 - R_2 \geq 0 \langle\langle R$, where R_1, R_2 are the radii of cup (sleeve) 1 and head 2, respectively. The load N' generates contact pressures p_α in the contact area described by an angle $2\alpha_0$. Elastic properties of the implant cup and head materials differ. The modulus of elasticity of materials in the case of MoP (metal-on-UHMWPE) endoprostheses differs by 450 ... 165 times, and in the case of CoP (ceramic-on-UHMWPE) endoprostheses - by 750 ... 270 times depending on the Young's modulus of UHMWPE (0.5 . .. 1.4 GPa). Such hybrid combinations of materials will be characterized by significantly different stresses and contact conditions than materials having similar elastic characteristics (metal-on-metal (MoM) and ceramics-on-ceramics (CoC) implants).

(a)

1 – prosthesis pin, 2 – femoral head, 3 – acetabulum cup, 4 – replacement hip socket



(b)

Fig. 1. Hip joint implant (a) and (b) calculation scheme for a cylindrical joint model [23].

3. PROBLEM SOLUTION METHOD

A solution of the analyzed contact problem requires the determination of maximum contact pressures, contact angle and – if necessary – the distribution of pressures in the contact area.

The equation of contact pressures p_α for the analyzed external loading system is given in [19,20]

$$c_1 \int_{-\alpha_0}^{\alpha_0} \cot\frac{\alpha-\theta}{2} p'_\theta d\theta = c_2 p_\alpha + c_3 \int_{-\alpha_0}^{\alpha_0} p_\alpha d\alpha + c_4 \cos\alpha \int_{-\alpha_0}^{\alpha_0} p_\alpha \cos\alpha d\alpha + \frac{\varepsilon}{R^2}, \tag{1}$$

where $p'_\theta = dp/d\theta$; α – polar angle; $0 \le \alpha \le \theta,\ -\alpha_0 \le \theta \le \alpha_0$;

$$c_1 = \frac{1}{8\pi}\left(\frac{1+\kappa_1}{G_1R_1} + \frac{1+\kappa_2}{G_2R_2}\right);\ c_2 = \frac{1}{4}\left(\frac{1-\kappa_1}{G_1R_1} - \frac{1-\kappa_2}{G_2R_2}\right);$$

$$c_3 = \frac{1+\kappa_1}{8\pi G_1R_1};\ c_4 = \frac{1}{2\pi}\left(\frac{\kappa_1}{G_1R_1} + \frac{1}{G_2R_2}\right);$$

are generalized characteristics of elasticity; G_1, G_2 are the shear moduli of the system component materials; ν_1, ν_2 are Poisson's ratios of the system component materials; $\kappa = 3 - 4\nu$ is the two-dimensional state of strain; $R_1 \approx R_2 = R$.

An approximate solution of Eq. (1) is obtained with the collocation method. A contact pressure function p_α was selected as [19,20]:

$$p_\alpha \approx E_0\varepsilon\sqrt{\tan^2\frac{\alpha_0}{2} - \tan^2\frac{\alpha}{2}}, \tag{2}$$

where the coefficient of collocation

$$E_0 = \left(e_4 / R^*\right)\cos^2\left(\alpha_0 / 4\right) \quad \text{or} \tag{3}$$

$$E_0 = \frac{e_4}{R^*}\left[\begin{matrix} \cos^{-2}\frac{\alpha_0}{4} - e_1\sqrt{\tan^2\frac{\alpha_0}{2} - \tan^2\frac{\alpha_0}{4}} - \\ 0.5\sin^2\frac{\alpha_{0\delta}}{4}\left(e_2\cos^{-1}\frac{\alpha_0}{2} + 2e_3\cos\frac{\alpha_0}{2}\right)\end{matrix}\right]^{-1}, \tag{4}$$

$$e_1 = \frac{2}{Z}\left[(1-\kappa_1)(1+\mu_1)E_2 - (1-\kappa_2)(1+\mu_2)E_1\right],$$
$$e_2 = \frac{2}{Z}(1+\kappa_1)(1+\mu_1)E_2,$$
$$e_3 = \frac{4}{Z}\left[\kappa_1(1+\mu_1)E_2 + (1+\mu_2)E_1\right],$$
$$e_4 = 4E_1E_2 / Z,$$
$$Z = (1+\kappa_1)(1+\nu_1)E_2 + (1+\kappa_2)(1+\nu_2)E_1,$$

$E = 2G(1+\nu)$ - Young's modulus of the material,

R^* - effective radius.

The maximum contact pressures p_0 are generated when $\alpha = 0$. Hence

$$p_0 \approx E_0\varepsilon\tan\frac{\alpha_0}{2}. \tag{5}$$

The half-angle of contact α_0 is calculated from the condition of equilibrium of forces acting on head 2 in the following way:

$$N' = R\int_{-\alpha_0}^{\alpha_0} p_\alpha \cos\alpha d\alpha = 4\pi R E_0 \varepsilon \sin^2\frac{\alpha_0}{4}. \tag{6}$$

To validate results obtained by the proposed analytical method, this problem was also solved by FEM in the ANSYS environment. According to the adopted calculation scheme in Fig. 1b, the FEM model of contact between cup 1 and head 2 was defined in the ANSYS preprocessor without friction (Fig. 1b). Given the significant difference between the elastic properties of the cup and head materials, particularly their Young moduli, in the developed calculation model the cup is a contact surface and the head surface is a target surface. The FEM model of the 2D system contact was defined with the use of 8-node PLANE183 finite elements that ensure an adequate accuracy when using irregular meshes.

The 2D model of the analyzed tribomechanical system was created using between 1606 and 3534 finite elements, depending on the component geometry. The 2D contact between the cup and the head was modelled using the elements CONTA172 (cup) and TARGE169 (head). In the 3D contact problem solution, the FEM mesh was generated using 20-node SOLID186 elements. The 3D models were developed using from 51816 to 62909 finite elements, depending on the system component dimensions. The 3D contact surface is modelled using the CONTA174 elements while the head surface is modelled with the use of three-dimensional elements TARGE170.

4. SOLUTION OF THE PROBLEM AND RESULTS

According to the literature [25,26], the total load N on the hip implant head consists of a body weight vector sum K and a muscular load M. This load significantly changes during physiological motion in the range of $-1.45K \le N \le 4.4K$. For K = 700 N, its mean value is approx. 1900 N and the maximum value is approx. 2900 N.

The following data were used in the calculation of hip implant contact parameters: $N = N_{sr}$ = 1900 N; D_2 = 28, 38, 48, 58 mm; $\varepsilon = 0.2$, 0.1 mm.

The implant components are made of the following materials: femoral head 2 – thermo-diffusion nitrided (TDN) titanium GRADE 2 [27], E_2 = 112 GPa, ν_2 = 0.32; cup 1 – UHMWPE, E_1 = 0.625 GPa (37°C), ν_1 = 0.46.

The modelling of the 2D cylindrical joint replacing the 3D implant system with the radii $R_1 \approx R_2 = R$ (Fig. 1b) included the introduction of a conventional radius $R^* = 0.5\sqrt{R_1 R_2} = 0.5\sqrt{(R_2+\varepsilon)R_2}$. It was established from the analysis of the contact geometry of bodies of double curvature. The condition is the equality of the contact areas of two spheres and two cylinders of similar diameters when they are in nearly conformal contact.

Results obtained with both methods (analytical and numerical) are given in Table 1 and in Figures 2 and 3. Maximum reduced van Mises stresses were also determined by the numerical method. Figures 2 and 3 show selected numerical results of contact pressures regarding their values and distributions.

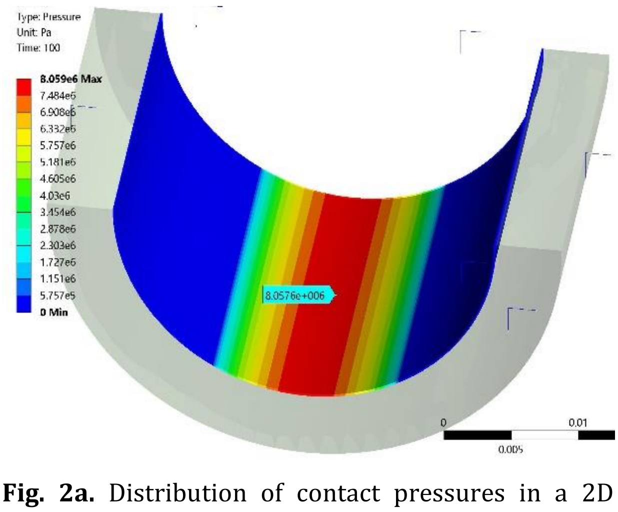


Fig. 2a. Distribution of contact pressures in a 2D system when D_2 = 28 mm, $\varepsilon = 0.2$ mm.

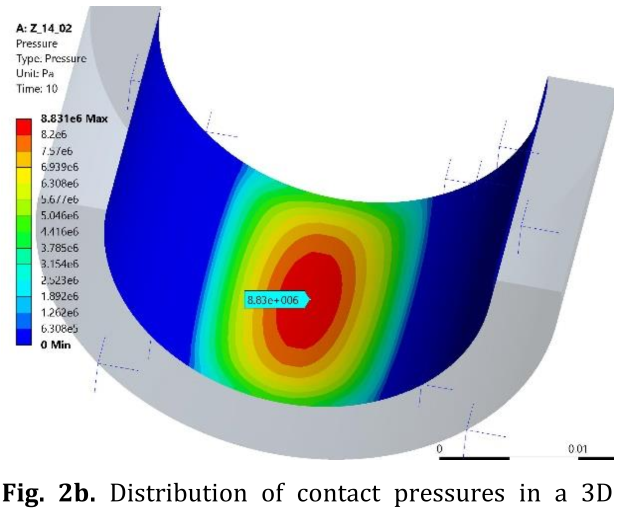


Fig. 2b. Distribution of contact pressures in a 3D system when D_2 = 28 mm, $\varepsilon = 0.2$ mm.

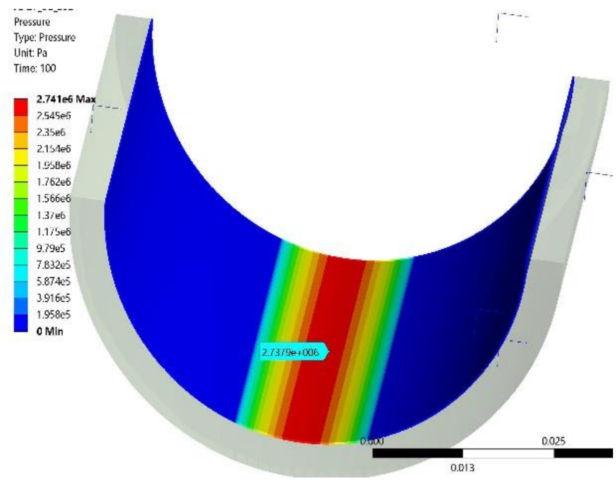


Fig. 3a. Distribution of contact pressures in a 2D system when D_2 = 58 mm, $\varepsilon = 0.2$ mm.

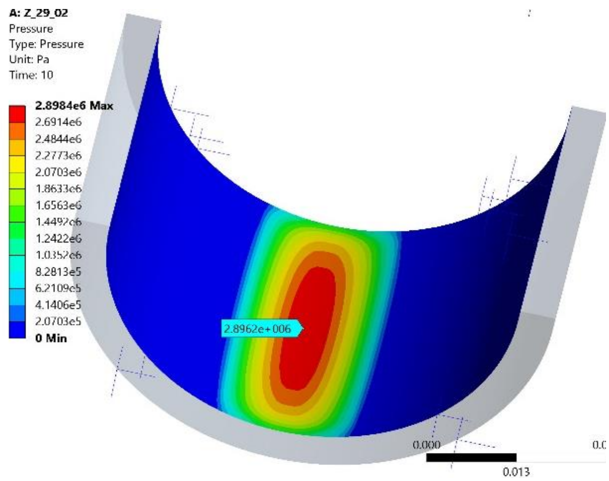


Fig. 3b. Distribution of contact pressures in a 3D system when D_2 = 58 mm, $\varepsilon = 0.2$ mm.

Table 1. Maximum contact pressures.

D_2, mm	Parameter	Radial clearance ε, mm	Radial clearance ε, mm
		0.2	0.1
28 S=5 mm	p_0, MPa	8.67 (8.82)	6.42 (6.71)
	$p_{\max}^{(2D)}$, MPa	8.06 [+7.0; + 8.6%]	6.06 [+ 5.6; +9.7%]
	$p_{\max}^{(3D)}$, MPa	8.83 [-1.8; -0.1%]	6.63 [-3.3; +1.2%]
	Mises, MPa	7.7	6.46
38 S=5 mm	p_0, MPa	5.42 (5.48)	3.95 (4.10)
	$p_{\max}^{(2D)}$, MPa	5.12 [+5.5, +7.0%]	3.77 [+ 4.6, +8.0%]
	$p_{\max}^{(3D)}$, MPa	5.61 [-3.5; -2.4%]	4.13 [-4.6; -0.7%]
	Mises, MPa	4.81	3.97
48 S=5 mm	p_0, MPa	3.8 (3.83)	2.75 (2.83)
	$p_{\max}^{(2D)}$, MPa	3.6 [+5.2; +6.0%]	2.71 [+1.5; +3.0%]
	$p_{\max}^{(3D)}$, MPa	4.03 [-6.1; -5.2%]	2.9 [-5.4; -2.5%]
	Mises, MPa	4.46	2.84
58 S=5 mm	p_0, MPa	2.85 (2.86)	2.05 (2.10)
	$p_{\max}^{(2D)}$, MPa	2.74 [+3.9; +4.2%]	2.03 [+0.98; +3.3%]
	$p_{\max}^{(3D)}$, MPa	2.89 (-1.4; -1.05%]	2.1 [-2.4; 0.0%]
	Mises, MPa	2.59	2.1

Notes: The indicated convergence (%) in the analytical results obtained with Eq. (5) and E_0 according to Eq. (3) in relation to the numerical results according to ANSYS; numerical results is given, compared; the pressures p_0 given in brackets were calculated for E_0 according to Eq. (4); in square brackets, the deviations of the pressures $p_{\max}^{(2D)}$, $p_{\max}^{(3D)}$ are indicated, in relation to the pressures p_0 calculated coefficient of collocation E_0 according to Eq. (3) and (4). Thickness of cup 1 (UHMWPE) S = 5 mm when calculated by ANSYS.

An analysis of the FEM results and those obtained with the developed calculation method show a good agreement. A comparison of the 2D and 3D problem solutions reveals that they considerably depend on the boundary effects causing changes in the contact surface and contact pressures. These effects cause a reduction in the contact area thus leading to higher contact pressures, which agrees with the results reported in [28,29]. An analysis of the maximum contact pressures reveals that the maximum contact pressures $p_{\max}^{(2D)}$ can be treated as the lower limit while the pressures $p_{\max}^{(3D)}$ as their upper limit (Fig. 4).

Table 2. Maximum contact pressures (analytical and according to literature data).

Authors	p_0 / p_{max}, MPa	p_0 / p_{max}, percent	Hip implant parameters
[4] S=8 mm	10.51 / 11.0 10.7 / 11.0	-4.7 -2.8	N = 2330 N; D_2 = 28 mm; ε = 0.1 mm; CoCr – UMHWPE
[5] S=15 mm	13.0 / 12.66 12.81/ 12.66	+2.6 +1.2	N = 3000 N; D_2 = 32 mm; ε = 0.23 mm; CoCr – UMHWPE
[7] S=8.7 mm	15.24 /17.1 16.07/17.1	-10.9 -6.0	N = 2500 N; D_2 = 22.225 mm; ε = 0.1825 mm; Stainless steel – UMHWPE
[8] S=9 mm	9.99 / 10.75 10.52 / 10.75	-7.6 -2.2	N = 2500 N; D_2 = 36 mm; ε = 0.3 mm; Ti – UMHWPE
[9] S= 10 mm	2.8 / 3.24 2.79 / 3.24	-15.7 -15.8	N = 2650 N; D_2 = 58 mm; ε = 0.1 mm; Ti – UMHWPE
	3.88 / 4.31 3.91 / 4.31	-11.1 -10.2	N = 2650 N; D_2 = 58 mm; ε = 0.2 mm; Ti – UMHWPE
	8.77 / 10.65 8.9 / 10.65	-21.4 -19.7	N = 2650 N; D_2 = 28 mm; ε = 0.1 mm; Ti – UMHWPE
	11.81 / 13.75 11.91 / 13.75	-16.4 -15.4	N = 2650 N; D_2 = 28 mm; ε = 0.2 mm; Ti – UMHWPE
[10] S=9.423 mm	8.84 / 8.5 8.34 / 8.5	-0.7 -1.9	D_2 = 32 mm; ε = 0.098 mm; CoCrMo – UMHWPE N = 1900 N
	9.07 / 8.9 8.56 / 8.9	+1.9 -4.0	N = 2000 N
	10.2 / 10.2 9.64 / 10.2	0.0 -5.8	N = 2500 N
	11.04 / 11.25 10.65 / 11.25	-1.9 -5.6	N = 2900 N
[11] S= 8 mm	11.95 / 12.73 12.03 / 12.73	-6.1 -5.5	N = 3000 N; D_2 = 32 mm; ε = 0.15 mm; CoCr – UMHWPE
[14] S= 7 mm	13.64 / 14.87 14.15 / 14.87	-9.0 -5.1	N = 2500 N; D_2 = 28 mm; ε = 0.25 mm; Ti – UMHWPE

Notes: E_0 according to Eq. (3) – upper line, E_0 according to Eq. (4) – lower line; Ti - GRADE 2; CoCrMo, CoCr – alloys.

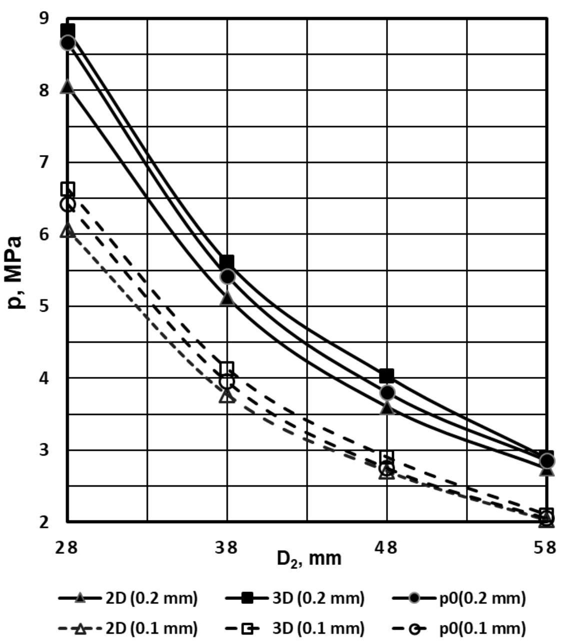


Fig. 4. Computational and numerical estimation of maximum contact pressures versus implant head diameter change (according to (3)).

With an increase in the diameter D_2 femoral head of the head by a factor of 2.07, the maximum contact pressures p_0 decrease non-linearly: at ε = 0.1 mm by a factor of 3.13 - Eq. (3), 3.2 times - Eq. (4); at ε = 0.2 mm in 3.04 times - Eq. (3) , 3.08 times - Eq. (4).

It is difficult to establish clear-cut trends between the numerical and computational results.

At higher radial clearance the results show a worse agreement in ANSYS 2D than in ANSYS 3D.

An inverse trend can be observed when the radial clearance is lower. With increasing the head diameter, the calculation results show more agreement with the 2D and 3D numerical results. It should be emphasized that the values of p_0 determined in accordance with the more precise Eq. (4) describing the collocation coefficient E_0 are higher by 0.35 ... 1.75% (0.2 mm), 2.44 ...

4.52% (0.1 mm) higher compared to those calculated according to Eq. (3). In 2D modelling according to ANSYS, increasing the head diameter reduces the deviation p_0 from $p_{\max}^{(2D)}$ (Table 1). However, in 3D modelling, the opposite tendency occurs (except when D_2 = 58 mm).

Since hip joint endoprostheses with cup thickness S < 5 mm [29] are known, the discrepancy between pressures p_0 and those set by ANSYS $p_{\max}$ will increase. This will limit the application of the developed method for estimating contact pressures in endoprostheses with a small thickness of the cup. At thicknesses S > 5 mm, a decrease in the pressure difference should be expected.

Below are given the results obtained with the proposed calculation method for different variants of the hip joint implant with the parameters given in [4,5,7-11] (FEM). Obtained results of the maximum contact pressures p_0 are compared with those reported in the afore-mentioned studies in Table 2.

An analysis of the data in Tables 1, 2 shows a good agreement between the results obtained by the proposed method and the FEM results reported in [4-8,10-11]. However, a significant difference can be observed between the results obtained by the developed calculation method and those reported in [9]. The discrepancy between the results of the analytical solution [13,14] and the author's method is not very significant.

Therefore, it should be expected that at $S \approx 7...8$ mm the contact stresses will not be significantly different than those obtained from the solution by the above method. A similar conclusion is also indicated in [17].

5. DISCUSSION

An analytical method of contact mechanics of the elasticity theory for calculation of maximum contact pressures in hip joint endoprostheses is developed. The contact of the head and the acetabular cup (ball-in-socked) in this spherical sliding hinge is modeled by a cylindrical hinge (cylinder-in-bushing) by applying the appropriate conditional (effective) radius. In the following, the classical contact problem of the elasticity theory about the internal contact of cylindrical bodies of close radii is considered and the method of its solution is given. It should be noted that in the literature there are no such specific approaches to the study of spherical hinges, in particular MoP endoprostheses.

The few simplified and approximate solutions known in the literature [12-14,16,17] use different approaches known in mechanics, in particular the Hertz method with the ball-in-plane contact model or the Winkler elastic base model. In particular, such a solution of the contact problem for the MoP of the hip joint endoprosthesis using the Hertzian theory, using the ball-in-plane model with the Hertzian conditional radius instead of the ball-in-socked model is given in [13]. Such an approach, however, should be considered simplistic given the significant difference in body contact models than the one proposed in this article. Winkler's model is used in works [16,17] and it is more suitable for almost conformal ball-in-socked contact.

Comparison of the results of calculations of the maximum contact pressures with the known in the literature obtained FEM, indicate their good or sufficiently satisfactory convergence (Table 2). Other authors did not compare the obtained FEM results with the analytical results of Jin [14].

Since, as noted, the convergence of the results presented in [4,5,7-11,14] with the author's is different, we used FEM (ANSYS 2D and 3D environment) to solve this problem and determine the maximum contact pressures for different head diameters and radial clearance. The good convergence of the results by both methods was confirmed, especially when using the hinge model in ANSYS 3D and the refined collocation coefficient in the analytical method. Sufficient convergence of the results of calculation of contact pressures by the proposed method with the results of studies available in the literature, both FEM and analytical [14], was also confirmed.

6. CONCLUSIONS

To estimate the maximum contact pressures in MoP hip arthroplasty, an analytical method was proposed, which is based on the classical method of studying the mechanics of internal contact of cylindrical bodies with close radii.

1. In order to compare the results of the calculations by the proposed analytical method, this problem was solved in the ANSYS environment. It has been established that the relative change in the results of FEM of the maximum contact pressure calculations in relation to the analytical results is:

   $p_0/\ p_{\max}^{(2D)} > 1.01...1.094$ times - Eq. (3);

   $p_0/\ p_{\max}^{(2D)} > 1.044...1.107$ times - Eq. (4);

   $p_0/\ p_{\max}^{(3D)} < 1.014...1.06$ times - Eq. (3);

   $p_0/\ p_{\max}^{(3D)} < 0.0...1.052$ times - Eq. (4).

2. The results of analytical deductions were also associated with the results of calculations using the finite element method known in the literature. The discrepancy of the results is quite varied for different parameters of their load, the diameters of the head and its materials and radial clearances (Table 2).

3. It is established that at $S \approx 7...8$ mm the contact stresses will not be significantly different than those obtained from the solution by the above method.

4. The proposed analytical method can be effectively used to calculate contact parameters, particularly maximum contact pressures in metal–on–polymer hip joint implants. The developed method can be effectively used to analyze both soft-on-hard (MoP, CoP) and hard-on-hard (MoM, CoC) implants, without significant transformation.

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