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RESEARCH

Optimizing Plant Oil-Derived Lubricants: A Sustainable Alternative to Petroleum-Based Lubricants Using Integer Programming

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A B S T R A C T

The depletion of fossil-based resources and growing environmental concerns have intensified the search for sustainable alternatives in lubrication. Mineral-based lubricants are becoming increasingly costly and environmentally problematic, necessitating the adoption of plant oil-derived lubricants. These bio-based alternatives offer significant environmental and performance benefits, including biodegradability, non-toxicity, and a reduced carbon footprint, while demonstrating favorable viscosity, lubricity, and thermal stability. However, challenges such as oxidative degradation, production costs, and low-temperature fluidity hinder their widespread adoption. In this study, a comparative evaluation of 13 plant oils was conducted using key physicochemical properties and economic factors. An integer programming model was then employed to optimize the selection process. The integer programming method extends previous research by providing a structured, computationally efficient approach that ensures a single optimal solution rather than a ranked list of potential candidates. The results indicate that rapeseed oil is the most viable option, as it best satisfies critical performance criteria—exhibiting a kinematic viscosity of 45.6 mm^2/s at 40°C, a flash point of 513°C, and a cost of only 4.62 per liter—while meeting sustainability benchmarks. The resulting framework not only identifies an optimal bio-based lubricant candidate but also provides practical insights for industrial implementation. Industries can utilize this framework to streamline lubricant selection, ensuring that the chosen formulation meets stringent performance and cost requirements. Policymakers, in turn, can leverage these findings to support the development and adoption of green lubrication technologies. While the results of the optimization model are promising, the proposed lubricant requires further experimental validation under real-world operating conditions to fully assess its long-term performance and durability before widespread industrial application is recommended.

1. INTRODUCTION

The future depends heavily on petroleum-derived products in the transportation and industrial sectors, which could raise significant concerns about future energy security. Plant oil-derived lubricants have the potential to replace the mineral-based lubricants.

While mineral oil-derived lubricants have been widely used due to their availability and established production processes, they present several significant challenges that have driven the search for sustainable alternatives, such as plant oil-based lubricants. Mineral oils, derived from non-renewable fossil resources, pose significant environmental challenges throughout their lifecycle, including extraction, refining, and disposal. Their non-biodegradable nature results in persistent pollution of soil and aquatic ecosystems, causing long-term environmental harm. In contrast, plant oil-based lubricants are inherently biodegradable, offering a substantially reduced environmental footprint. Furthermore, mineral oils frequently contain toxic additives and by-products, which can be harmful to both human health and wildlife. Their application in sensitive environments, such as agricultural or marine settings, exacerbates ecological risks. Plant oils, however, are non-toxic and environmentally benign, making them a safer alternative for both users and ecosystems.

The production and utilization of mineral oils are major contributors to greenhouse gas emissions, thereby accelerating climate change. In comparison, plant oils, sourced from renewable resources, exhibit a significantly lower carbon footprint, aligning with global efforts to achieve sustainability goals. The finite nature of fossil resources further underscores the unsustainability of relying on mineral oils, whereas plant oils can be continuously replenished through sustainable agricultural practices. Although mineral oils have been extensively optimized for various industrial applications, their performance often depends on the addition of chemical additives, such as anti-wear or antioxidant agents. These additives not only increase costs but also amplify the environmental impact. In contrast, plant oils possess inherent properties—such as excellent lubricity, viscosity, and thermal stability—that minimize the need for supplementary additives, as demonstrated in our study. Additionally, the price volatility of mineral oils, driven by geopolitical factors and the depletion of finite resources, poses economic challenges. Plant oils, on the other hand, can be locally produced, providing greater economic stability and reducing reliance on volatile global oil markets. Depending on the main outcomes, it can be argued that plant oil-derived lubricant is a favorable replacement for diverse implementations because of their usability in the large series of features which are necessary for several implementations.

In summary, the increasing depletion of fossil-based resources and growing environmental concerns highlight the urgent need for sustainable alternatives in the lubrication industry. While mineral-based lubricants are effective, they contribute to environmental pollution, exhibit limited biodegradability, and are subject to volatile market prices. Plant oil-derived lubricants present a promising solution due to their renewability, non-toxicity, and favorable tribological properties. However, their widespread adoption is hindered by critical challenges such as oxidative instability, poor low-temperature performance, and high production costs.

To address these limitations, this study employs an integer programming (IP) model to optimize lubricant selection. Unlike traditional heuristic or multi-criteria decision-making methods, IP can systematically balance competing objectives—like cost, performance, and sustainability—while adhering to real-world production constraints. The model evaluates 13 plant oil alternatives based on key physicochemical properties (e.g., viscosity, flash point, pour point) and economic feasibility to determine the most viable lubricant option. The plant oil-derived lubricants indicate preferable lubricant parameters over the traditional mineral-sourced lubricants, with biodegradability and renewability being their greatest strengths. Triglyceride oils have preferable qualifications than mineral-based lubricants as they benefit from a big lubrication energy, friction-wear resistance, high viscosity index, biodegradability, and renewability [1,2]. For lubrication, the good pour point and less oxidation steadiness of triglyceride oils have restricted their direct utilization. Chemical alteration of triglyceride oils is the most encouraging route to develop the unwanted restrictions of triglyceride oils for lubricant implementation [1,3]. Despite their benefits, widespread use of bio-derived lubricants is still limited today due to key barriers related to production scale, efficiency and lack of incentives from authorities. Sundry bio-sourced lubricants, especially raw plant oils have subnormal low temperature and oxidative stability parameters. Nonetheless, such defects can be overcome by the addition of proper additives and the suitable chemical modifications [4]. The feedstock's cost is one of the elements that limits the bio-lubricant generation since it takes up approximately 75 percent of the whole cost. It is worth noting that the expense of bio-based lubricants is typically between 5 and 10 times more than that of mineral-sourced oils [5]. Other elements, like water sources, present mechanization, climate for fertile agricultural areas, cultivation, and energetic human resources are the efficient and restrictive elements in selection oil bearing bio-based product as the bio lubricant raw material [6,7]. Sustainability policies towards the application of bio-based lubricants are not only focused on ecological topics but the sustainable feedstock's selection should also be thought. For this reason, detailed research on the latest perspectives and progress on the bio lubricant generation potential from oil-bearing biomass sources in the world is requisite.

Lubricants are used to prevent heat loss and wear between contacting surfaces and to protect them from shrinkage, oxidation and deterioration. They also act as protective media in the transformer's conformity and act as preventive agents of dust, water and dirt [8]. Generally, plant oil-derived lubricants (chemically modified or pure plant oils) are obtained to possess excellent lubricity than petroleum-sourced oil, especially under limited lubrication conditions, as recorded in a few research studies [9]. This is generally due to the fact that fatty acid molecules within the plant oils can chemically react with metal superficies to shape less shear resistance metallic soap sheet. An instance of this elementary reaction would be the iron stearate sheet's foundation the stearic acid and iron's reaction. The metallic soap sheet (iron stearate) will efficiently decrease the friction coefficient among superficies [10]. Oil including chained long fatty acids (C18 and over) displayed better lubricity and anti-wear features in comparison to oils including shorter chain length of fatty acids. This is because of plant oils having high percentage of oleic acid are able to sustain lower friction coefficient and less abrasive because of oleic acid kinds intense fatty acid mono-sheet that can reduce the asperity protects and contact the metallic superficies in the course of a running [11]. Nevertheless, raise in ratio of abrasive on slippery interfaces was recorded once utilizing bio-sourced lubricants. The continual slippery move will conclude in metallic soap layer's continuous removal. The metallic soap layer reconversion is again through the identical chemical treatment, emerging raise in wear. The increase in abrasive is also conducted through the peroxides' corrosive impact and free fatty acids as oxidation's products [12]. For boundary lubrication, Zulkifli and his coworkers noted that raising condensation of palm-based oil ester raised the abrasive on steel superficies. It was explicated that at high temperature, bio-sourced lubricant can be leaving the corrosive materials sheet on superficies and simply oxidize [13]. Once utilized as clean lubricant, chemically formed plant oils are more favorable than raw plant oil especially in relation to wear and friction. This is because of chemically modified plant oils having greater viscosity than their unchanged peer, activating the fluid film's structure more efficiently in the course of process. Chemically modified plant oils also have excellent oxidative steadiness that significantly decrease the oxidation products' structure (free peroxides and fatty acids), thus decreasing the total amount of wear. Adhvaryu et al. argued that chemically formed soybean oil has shown lower than half the coefficient of friction once compared to raw soybean oil [14]. It was suggested that the contrasting physical classes in the triacylglycerol molecule interact chemically and physically with metallic surfaces under sliding contact and high load. Thus, the polar functionality's increase in the plant oil formation by chemical alteration has an affirmative effect on lubricity [14,15].

The primary objective of this research is to identify the most feasible and sustainable plant oil-derived lubricant through a rigorous optimization framework. By integrating empirical data with mathematical modeling, the study aims to provide a quantitative decision-making tool for researchers, engineers, and policymakers. Ultimately, the findings will not only guide the selection of optimal bio-based lubricants but also pave the way for future advancements in green lubrication technologies through chemical modifications and additive enhancements.

This paper provides a research on bio-sourced lubricants produced by the common types of plant oil raw materials. Secondly, comparative evaluation of thirteen different sustainable plant oil alternatives are provided. Using mathematical modeling and integer programming approach, selection of vegetable oils is investigated and classified according to general important properties. The aim of this study is to select the most applicable and sustainable vegetable oil as lubricant using mathematical modeling.

2. REVIEW OF LITERATURE

2.1. The plant oil as raw materials for bio-derived lubricants

Plant oils, derived from seeds, plants, and fruits, have emerged as promising raw materials for bio-based lubricants due to their non-toxic, renewable, and biodegradable properties [16–20]. However, their susceptibility to oxidative and thermal degradation, attributed to the presence of methylene-interrupted double bonds, limits their practical application. These chemical structures lead to suboptimal characteristics, including poor oxidative stability, limited thermal performance, and undesirable sensory traits in their natural form [21–23]. Despite these challenges, plant oils predominantly consist of triglycerides (approximately 97%), alongside minor components such as phytosterols, phospholipids, and fat-soluble vitamins, which contribute to their functional properties [24,25]. Recent studies have demonstrated the potential of plant oils as base oils for lubricants, with modifications enhancing their performance [26].

For instance, research conducted in Korea investigated the oxidative and thermal stability of various plant-derived oils, including olive, sesame, sunflower, perilla, canola, and rice bran oils, under high-temperature conditions (up to 300°C). The findings revealed significant thermal oxidation in these oils, highlighting the need for stabilization methods to improve their performance [26]. Similarly, a comparative study of camellia, peanut, and palm oils under controlled heating conditions demonstrated that camellia oil exhibited superior oxidative stability and minimal changes in fatty acid composition compared to peanut and palm oils [27]. In the Moroccan market, the oxidative stability of palm, sunflower, and rapeseed oils was evaluated at elevated temperatures (up to 180°C for 30 hours). Palm oil was found to be the most stable, further underscoring the variability in performance among plant oils [28].

Moreover, the oxidative and thermal stability of cottonseed, palm, soybean, sunflower, canola, and corn oils was analyzed with and without antioxidants for use as metal quenchants. Using pressure-differential scanning calorimetry, palm oil exhibited the highest stability, while antioxidants significantly improved the performance of both palm and soybean oils [28]. Another study compared the oxidative-thermal stability of sunflower, sesame, and coconut oils, concluding that sesame oil offers superior friction reduction, thermal stability, and wear resistance. However, its oxidative stability requires further enhancement [29].

While these studies provide valuable insights into the properties of plant oils, they also reveal critical research gaps. For example, the majority of existing research focuses on individual oils under specific conditions, with limited comparative analyses across a broader range of temperatures and applications. Additionally, the mechanisms underlying oxidative degradation and the effectiveness of stabilization methods, such as chemical modification and antioxidant addition, remain underexplored. These gaps highlight the need for systematic studies to optimize plant oil-based lubricants for diverse industrial applications.

In summary, the primary limitations of plant oil-derived lubricants—poor low-temperature performance and oxidative instability—can be addressed through various strategies, including the addition of antioxidants, chemical modification, and emulsification. Among these, chemical modification shows particular promise in enhancing the oxidative stability and physical properties of plant oils across a wide temperature range.

Previous research on lubricant selection has explored various aspects, ranging from consumer behavior to technical evaluations, yet these studies often lack a comprehensive and systematic approach. For instance, one study emphasizes the importance of understanding oil labeling systems and the implications of reduced ZDDP (zinc dialkyldithiophosphate) levels in modern oils, particularly for classic car engines. While this approach is valuable for raising consumer awareness, it relies heavily on summarizing technical data and providing practical recommendations, targeting a non-specialist audience. As a result, it lacks a systematic framework for optimizing lubricant selection based on quantitative performance criteria [30]. Another study investigates the awareness and practices of vehicle owners regarding engine oil selection and maintenance, revealing significant gaps in consumer knowledge. For example, the majority of respondents rely on mileage for oil change schedules, with only a small percentage considering time-based intervals. Additionally, the study highlights a lack of understanding about the technical aspects of lubrication systems and the importance of oil properties. While this research provides valuable insights into consumer behavior, its methodology is limited to descriptive survey analysis and does not offer a systematic framework for optimizing lubricant selection or addressing technical performance criteria [31].

Further research critiques the subjective nature of lubricant selection, highlighting widespread misconceptions and personal biases, such as brand loyalty and viscosity preferences. For instance, some users claim satisfactory performance with oils that allow 100,000 km between overhauls, while others emphasize the importance of oil quality and performance under pressure, expecting a minimum of 500,000 km. The study also notes the lack of specificity in oil recommendations, pointing out that even within the same brand, different products can vary significantly in quality and performance. While this critique provides a thoughtful analysis of consumer behavior, it does not offer a systematic or data-driven methodology for lubricant evaluation. Instead, it relies on anecdotal evidence and general observations [32].

Recent advancements in lubricant selection methodologies include the use of Multi-Criteria Decision-Making (MCDM) approaches to evaluate lubricants for specific engine types. For example, one study employs a combined compromise solution method to assess lubricating oils for engines with higher performance requirements due to increased carbon monoxide and hydrocarbon emissions. The study evaluates four types of lubricating oils based on key parameters such as density, viscosity index, and viscosity at 100°C and 40°C, with criteria weights determined using entropy, CRITIC, and standard deviation methods. The results demonstrate that the optimal lubricant choice remains consistent across all weighting methods, providing a robust framework for decision-making. However, this study is limited to a narrow set of criteria (density and viscosity) and specific engine types, leaving broader applications and sustainability considerations unaddressed [33].

In contrast to these studies, our research employs a rigorous, data-driven methodology to evaluate and optimize plant oil-based lubricants. We conduct a comparative analysis of 13 plant oils based on key physicochemical properties (e.g., kinematic viscosity, flash point, oxidative stability) and economic factors, followed by an integer programming model to identify the most viable option. Unlike previous studies, which are either descriptive, survey-based, or limited to specific engine types, our approach provides a scientific and industrial framework for lubricant selection, addressing both performance and sustainability benchmarks. This methodological distinction highlights how our study advances previous work by offering a systematic, optimization-based solution tailored for industrial applications. Furthermore, our research integrates a broader range of lubricant properties and applications, ensuring a more comprehensive evaluation that incorporates sustainability considerations, which are often overlooked in earlier studies.

Recent studies have explored the potential of plant oils in various industrial applications, highlighting their versatility and sustainability. For instance, Balo and Yücel [34] assessed the thermal performance of green building materials produced with plant oils, demonstrating their suitability for energy-efficient construction. Similarly, Sagbansua and Balo [35] conducted a multi-criteria analysis to select epoxidized vegetable oils for insulation materials and plastic manufacturing, emphasizing the importance of optimizing material properties for specific applications. In another study, Sagbansua and Balo [36] formulated an integer programming model to identify the optimum renewable oil for biodiesel production, showcasing the applicability of optimization techniques in sustainable energy solutions. These studies collectively underscore the growing interest in plant oils as eco-friendly alternatives across diverse industries, paving the way for further research and innovation. Important features of bio oil-derived lubricants for different implementations are displayed in Table 1 [37].

Table 1. Important features of bio oil-derived lubricant for different implementations.

Implementation	Lubricant	Features	Remarks
Practicable to overall implementations	plant oils and diverse raw materials' natural ester	renewable, good bio-degradability, broad viscosity ratio, good lubricity, low volatility, good thermal stability, good viscosity index, proper oxidation steadiness	solely displayed through derivatives of chemically modified plant oil, proper for different of implementations, eco-friendly, proper for high-temperature implementations
Hydraulic oil	rapeseed oil	quick air discharge ratio	lower noise and vibration
	high oleic soybean oil	less compressibility	good pressure transmission
Engine oil	rapeseed oil, Karanja oil	better lubricity	fix brake power and engine efficiency
	palm oil, castor oil	less evaporating natural combination emission	minimized engine emission such as HC, CO
Insulating fluid	plant oil	good water resolubility grade	diminish the humidity effect on insulating strength
Chainsaw oil	plant oil	less volatility	less danger mist production
Compressor oil	epoxidized soybean oil	good thermal stability	can endure the norm compressor release temperature (about 250 ^{0}C)
Gear oil	soybean, cottonseed, corn, rapeseed	better dielectric coefficient	Better insulating features
	polyethylene tetra-oleate ester	better weld load	
	complicated esters	better lubricity	by the adding of proper additives
Metalworking lubricant	soybean oil emulsions	good antirust capability	high tool life
		less volatility	less danger mist production
	palm oil, coconut oil	good lubricity	minimized superficies roughness, decreased cutting temperature, and high tool life
	rice bran, soybean, neem, Karanja,	good emulsifiability	stable emulsions at high temperature

2.2. Primary properties of analyzed plant oil-derived lubricants

The structure of saturated fatty acid and unsaturated fatty acid of plant oils that were analyzed are shown in Table 2 and Table 3, respectively [37]. The fatty acid composition of plant oils, as presented in Tables 2 and 3, plays a critical role in determining their suitability as bio-based lubricants. Saturated fatty acids (Table 2) contribute to the thermal stability and oxidative resistance of lubricants, as they are less prone to degradation at high temperatures. For instance, oils like palm and coconut, which exhibit high percentages of saturated fatty acids (e.g., C16:0 and C12:0), are more stable under extreme conditions but may suffer from poor low-temperature fluidity. On the other hand, unsaturated fatty acids (Table 3), particularly monounsaturated (e.g., C18:1) and polyunsaturated (e.g., C18:2, C18:3) acids, enhance the lubricity and biodegradability of plant oils. However, higher levels of unsaturation can increase susceptibility to oxidative degradation, as seen in oils like soybean and sunflower. The balance between saturated and unsaturated fatty acids is therefore crucial for optimizing the performance of plant-based lubricants. For example, rapeseed oil, with a balanced profile of C18:1 (64.7%) and moderate saturated acids (4.6% C16:0), demonstrates excellent lubricity and thermal stability, making it a strong candidate for industrial applications. These compositional insights directly influence the optimization model by providing a scientific basis for selecting oils that meet both performance and sustainability criteria. The unsaturation degree and the fatty acid's structural contents of plant oils are displayed in Table 4 and Table 5, respectively [37-43]. The unsaturation degree and elemental composition of plant oils, as detailed in Tables 4 and 5, are critical factors influencing their performance as bio-based lubricants. Table 4 presents the unsaturation degree in three forms: unweighted, partially weighted, and fully weighted, which reflect the number and position of double bonds in fatty acids. Oils with higher unsaturation degrees, such as sunflower (1.57 fully weighted) and soybean (1.51 fully weighted), exhibit superior lubricity and biodegradability due to the presence of multiple double bonds. However, these oils are also more prone to oxidative degradation, which can limit their thermal stability. In contrast, oils with lower unsaturation degrees, like coconut (0.12 fully weighted), demonstrate better oxidative stability but may lack sufficient lubricity for demanding applications. Table 5 complements this analysis by providing the elemental composition (carbon, hydrogen, and oxygen) of each oil, which directly impacts their energy content, viscosity, and thermal properties. For instance, oils with higher carbon content (e.g., rapeseed at 77.07% C) generally offer better energy efficiency, while the oxygen content influences biodegradability and compatibility with additives. The cost of plant oilseed at bio-lubricant production is given in Table 6 [44-46]. Table 7 presents key physicochemical properties of plant oil-derived lubricants, which are critical for evaluating their performance in industrial applications. [37-43]. Viscosity index (VI), kinematic viscosity (at 40°C and 100°C), density, acid value, pour point, and flash point are among the most important parameters. For instance, kinematic viscosity directly influences the lubricant's ability to form a protective film under varying temperatures, with higher values at 40°C (e.g., castor oil at 220.6 mm^2/s) indicating better load-bearing capacity but potentially poorer fluidity at low temperatures. The viscosity is the most important feature of lubricant oils. It signifies the interior friction within a fluid because of molecular interplay, which in return affects the capability of oil to create a lubricating film in the course of the process. The viscosity of most available raw plant oils varies in the range of 30-50 mm2s-1, excluding castor oil which has a viscosity value of about 220 mm2s-1 because of the high degree of long-chained fatty acids (about 95 percent). Bio-sourced lubricants have comparatively elevated viscosity value in comparison with fossil-sourced lubricants. This activates the lubricants to function in broader temperature intervals which is needed in specific implementations. The viscosity index, which reflects the oil's viscosity stability across temperature ranges, is particularly high in soybean oil (VI = 246), making it suitable for applications with wide temperature fluctuations. The pour point, a measure of low-temperature fluidity, varies significantly among oils, with coconut oil exhibiting the highest pour point (294 K), limiting its use in cold environments. Conversely, the flash point, which indicates thermal stability, is highest in soybean oil (598 K), making it safer for high-temperature applications. These properties collectively guide the selection of plant oils for specific industrial uses, ensuring that the chosen lubricant meets both performance and safety requirements. The comparison with the ISO VG46 reference lubricant further highlights the potential of plant oils to meet or exceed conventional standards, particularly in terms of viscosity and thermal stability.

Lubricant features at high temperature include flash point and volatility. The volatility value is the lubricant's tendency to evaporate whilst flash point value is the temperature point under which the vapor is flammable in the presence of air. A lubricant with elevate temperature features should have flash point and volatility above-mentioned the calculated running temperature. Generally, bio-sourced lubricants obtained from plant oils show greater flash point value (larger than 200^{0}C) when crosschecked to the main reference such as SAE20W-40. The high value of flash point also signifies the plant oils' non-volatile nature and can be certified to the fatty acids' presence within plant oils [38]. The lubricant features at low temperature are generally correlated with pour and cloud points. The cloud point value is the temperature under which the lubricant commences to become cloudy or hazy in feature. As the temperature drops, it is because of the wax's crystallization operation within the lubricant. The pour point value is the temperature below-mentioned cloud point value under which the lubricant discontinues to pour or flow. Lubricant utilized in engines or machineries running at low temperature should have lower temperature features to avoid clogging of filters. Because of saturated fatty acid's high amount (larger than 50 percent), palm and coconut oils have low pour point value, 21 and 23^{0}C, respectively. However, palm oil-derived 3methylol-propane ester shows significantly lower pour point (about -36^{0}C), showing that suitable chemical alteration can efficiently overcome lower temperature features of the raw plant oils.

Table 2. The structure of saturated fatty acid of plant oils analyzed.

Plant oil	Saturated fatty acids (%)								
	C8:0	C10:0	C12:0	C14:0	C16:0	C18:0	C20:0	C22:0	C24:0
Castor	–	–	–	–	1.30	1.20	–	–	–
Soybean	–	–	–	0.06	9.90	3.94	0.41	0.48	0.21
Olive	–	–	–	–	16.50	2.30	0.43	0.15	0.06
Sunflower	–	–	0.02	0.09	6.20	2.80	0.21	–	0.31
Rapeseed	–	0.01	–	–	4.60	1.70	–	–	–
Jatropha	-	-	-	-	6	-	-	-	-
Coconut	7.60	5.50	47.70	19.90	–	2.70	–	–	–
Rice bran	–	–	–	0.39	20.0	2.10	–	–	–
Palm	–	–	–	1.12	42.70	4.55	0.39	0.58	0.06
Cottonseed	–	–	–	0.77	21.87	2.27	0.26	0.36	0.12
Sesame	–	–	–	–	9.70	6.50	0.63	0.14	–
Moringa	-	-	-	-	6.5	72.2	2	-	-
Corn	–	–	–	–	10.34	2.04	0.44	0.31	0.26

Table 3. The structure of unsaturated fatty acid of plant oils analyzed.

Plant oil	Unsaturated fatty acids (%)									
	C16:1	C17:1	C18:1	C18:2	C18:3	C20:1	C20:2	C22:1	C22:2	C24:1
Castor	–	–	89.70	7.30	0.50	–	–	–	–	–
Soybean	0.08	0.08	21.35	56.02	7.15	0.22	–	–	–	–
Olive	1.80	–	66.4	16.40	1.60	0.30	–	–	0.05	–
Sunflower	0.12	–	28.0	62.2	0.16	0.18	0.09	–	–	0.39
Rapeseed	0.21	–	64.7	19.60	1.20	9.10	–	–	–	–
Jatropha	-	-	50	30.2	-	-	-	-	-	-
Coconut	–	–	6.20	1.60	–	–	–	–	–	–
Rice bran	0.19	–	42.70	33.1	0.45	1.11	0.11	–	–	0.38
Palm	–	0.06	39.37	10.62	0.21	0.17	–	–	–	0.06
Cottonseed	0.47	0.11	16.61	56.35	0.33	0.14	0.10	–	–	0.16
Sesame	0.11	–	41.5	40.9	0.21	0.32	–	–	–	–
Moringa	2	1	-	-	4	-	-	-	-	-
Corn	–	0.05	24.23	60.38	0.99	0.28	–	–	–	0.20

Table 4. The unsaturation degree of plant oils analyzed.

Vegetable oil	Unsaturation's 'unweighted' Degree (%)	Unsaturation's 'Partially weighted' degree [b]	Unsaturation's 'Fully weighted' degree [c] (double bonds' mean number)
Castor	0.90	1.01	1.01
Soybean	0.84	1.44	1.51
Olive	0.85	0.96	0.97
Sunflower	0.89	1.57	1.57
Rapeseed	0.93	1.24	1.32
Jatropha	0.78	1.15	1.15
Coconut	0.09	0.12	0.12
Rice bran	0.78	1.14	1.15
Palm	0.51	0.62	0.62
Cottonseed	0.71	1.27	1.27
Sesame	0. 79	1.37	1.59
Moringa	0.92	1.12	1.11
Corn	0.86	1.45	1.45

[a] Overall unsaturated fatty acids supposed to have the identify weight percent.
[b] Overall unsaturated XX: y (y-2) fatty acids have a weight percent of two.
[c] XX: three fatty acids a weight percent of 3, XX: 2 fatty acids have a weight percent of two and so on.

Table 5. The fatty acid's structural contents of plant oils analyzed.

Plant oil	C (% w/w)	H (% w/w)	O (% w/w)
Castor	76.57	11.94	11.42
Soybean	77.03	11.90	10.95
Olive	76.91	11.66	10.95
Sunflower	76.90	11.84	10.98
Rapeseed	77.07	11.84	10.93
Jatropha	76.57	12.21	11.32
Coconut	72.75	11.65	11.01
Rice bran	76.22	12.38	11.26
Palm	76.09	12.44	11.27
Cottonseed	76.86	11.89	11.21
Sesame	77.02	11.98	10.87
Moringa	76.16	11.73	11.83
Corn	76.71	11.52	10.98

Table 6. The cost of plant oils used at bio-lubricant production.

Plant oil	Castor	Soybean	Olive	Sunflower	Rapeseed	Jatropha	Coconut	Rice bran	Palm	Cottonseed	Sesame	Moringa	Corn
Cost (Per Liter)	12,15	3.99	20.00	3.35	4.62	197.00	19.21	195.00	1.22	2.65	74.64	209.00	12.00

The lubricants based on animal fats or plant oils are named bio-lubricants and are gaining recognition and adoption because of their environmentally friendly and sustainable features. CO_2 from the use of biolubricants does not eliminate any harm to the environment [32]. In the next 20 years, the environment-friendly lubricant market's share would rise to 15 percent and even upwards of 30 percent in several areas. Due to the bio lubricants' increasing implementation in the manufacturing and transportation sectors, this market is predicted to observe an important outgrowth [40]. The bio-sourced lubricants have displayed more preferable lubrication features than traditional synthetic and mineral oils and are able to ensure the necessary features required through different engineering implementations. Furthermore, because of their benefits of being non-toxic and clean, they have been commanded to be used in very susceptible implementations like marine and forestry by some countries' stringent regulations and laws. Due to these reasons, the requisitions for bio-sourced lubricants are predicted to rise in the future, together with their execution in other implementations such as hydraulic oils, engine oils, and so on. Growing cultivations of second-generation raw materials or plant oils, in the future will certainly aid in supplying sufficient feedstocks for continual bio-sourced lubricant fabrication [41-43].

Crops of oilseed are Calophyllum inophyllum, Jatropha curcas, Rubber seed (Hevea brasiliensis), Karanja (Pongamia pinnata), Jojoba (Simmondsia chinensis), and castor (Ricinus communis). Micro-algae which is thought as the third-generation raw material, has become the bio-sourced lubricant's recent potency resource. The microalgae were recorded to crop oil's higher amount (upwards of 70 percent) in comparison with other oil products like jatropha (28 percent), sunflower (40 percent), and palm (36 percent) [47].

Table 7. Physicochemical properties of analyzed plant oil-derived lubricants.

	Viscosity	[a]Density @ 15 °C	Kinematic Viscosity	Kinematic Viscosity	Acid value	Pour point	Flash point	Reference
	Index	kg/m³	@ 100 °C mm²/s	@ 40 °C mm²/s	mg KOH/g	K	K	
Castor	220	0.950-0.970	19.72	220.6	1.40	246	523	[49]
Soybean	246	0.922-0.934	7.55	28.86	0.30	264	598	[50]
Olive	190	0.914-0.925	8.24	39.62	1.10	270[b]	591[b]	[51]
Sunflower	203	0.920-0.927	8.65	40.05	0.30	261	525	[49]
Rapeseed	180	0.910-0.917	10.07	45.60	1.50	261	513	[49]
Jatropha	170	0.917	5.50	40.0	4.65	270	546	[52]
Coconut	169	0.919-0.937	5.5	24.8	-	294	598	[53]
Rice bran	201	-	8.7	40.6	-	260	591	[53]
Palm	-	-	-	39.4	0.50	297	525	[54]
Cottonseed	211	0.917-0.931	7.75	33.86	-		525	[55]
Sesame	193	-	6.3	27.33	2.00	268[b]	589[b]	[56]
Moringa	-	-	-	44.88	-		477	[57]
Corn	238	0.916	8.06	32.41	0.30	259[b]	597b	[58]
Reference Lubricant								
ISO VG46	>90	-	>4.1	>41.4	-	267	493	[59]
[a] [60] [b] [61]								

In terms of marketability, industrial consumers can only prefer bio-sourced lubricants as a cost-effective alternative to traditional petroleum-based lubricants. Although the relatively expensive first purchase expense can deter buyers of potential lubricants, the lower energy costs associated with bio-based lubricants' high lubricity are often offset. Below are some examples of the energy savings that bio-based lubricants can provide in various applications: compressors, 2-7 percent; hydraulics, 1-3 percent; plastics injection moldings equipment, 2-3 percent; spur gears, 1-5 percent, and worm gears, 15-30 percent [48]. Researchers should concentrate their efforts in the future on improving big-scale production performance in relation to cost-effectiveness and yield ratio. Improvement of reliable and cheaper raw materials by genetic engineering is key in supporting the bio-sourced lubricants' work. It is critical to evaluate methodologies for tribological and physio-chemical features for the final product. To maintain testing procedures and performance levels, testing organizations, end users, and lubricant suppliers must continue to collaborate. In determining the performance standards for environmentally friendly lubricants, suitable guidelines from organizations such as iso and astm are needed. Thorough research based on these criteria is needed to persuade operators and manufacturers on the benefits of bio-sourced lubricants.

3. MATHEMATICAL MODELING METHODOLOGY

This study involves the physicochemical properties of the selected plant oil-derived lubricants. The properties of plant oil-based lubricants are kinematic viscosity at 40°C, density at 15°C, kinematic viscosity, acid value at 100°C, pour point, flash point, and viscosity. 13 alternative lubricants are compared using integer programming method. The application of integer programming in lubricant selection has been explored in related optimization problems, particularly in supply chain management, resource allocation, and industrial material selection. Previous studies employing multi-criteria decision-making (MCDM) approaches, such as AHP and TOPSIS, have successfully ranked bio-based lubricants based on tribological properties. For instance, Hussain et al. (2022) utilized the TOPSIS MCDM model to rank vegetable oil-based biolubricants by assessing their performance and properties [62]. Ahmad et al. (2019) documented the use of a hybrid AHP-TOPSIS technique to analyze material formulations, demonstrating its effectiveness in ranking formulations based on performance measures, including tribological properties [63]. Additionally, Kathamore and Bachchhav (2020) focused on the grade classification of bio-based lube oils, applying multi-attribute decision-making methods to select appropriate lubricants for metal cutting operations while considering various performance metrics [64].

While these studies provide valuable insights into ranking bio-based lubricants, they often lack a structured optimization model that can simultaneously minimize cost while satisfying physicochemical constraints. The integer programming method extends previous research by providing a structured, computationally efficient approach that ensures a single optimal solution rather than a ranked list of potential candidates. The formulation for the optimization problem is presented below. Based on the nature of the variables and the limitation, the problem is designed as an integer problem (IP). Thus, the integer programming approach was used for the solution of the optimization problem. An integer programming problem is described as a modeling problem that includes binary, integer, or all types of restrictions. The integer restrictions make modeling nonconvex, and finding an optimum solution for an integer programming problem is the same as resolving a global optimization problem. Without the integer restrictions, such problem types can take much longer to solve.

Integer programming is particularly suitable for this study for a number of reasons. First of all, the selection of an optimal plant oil-derived lubricant involves discrete decision-making. The problem requires choosing a single lubricant from a set of alternatives based on multiple constraints, making binary integer programming (BIP) the most appropriate approach. Unlike continuous optimization methods, which allow fractional solutions, integer programming ensures that the decision variables take only binary values (0 or 1), meaning either a lubricant is selected (1) or not (0). Secondly, the selection must satisfy constraints on physicochemical properties (e.g., kinematic viscosity, pour point, flash point) while minimizing cost. Integer programming effectively handles such multi-criteria decision-making problems. Since bio-based lubricants are costly and subject to various production limitations, the model ensures the selection of an optimal solution while adhering to predefined constraints. The constraints and objective function in this problem lead to a non-convex optimization space, where integer programming methods (such as Branch and Bound) are effective in finding the global optimum. Finally, in real-world applications, only one type of lubricant is typically chosen for production or industrial use. Integer programming guarantees that the final output is a single, feasible solution rather than a mix of multiple alternatives. Since the nonlinear solution method is used, a Branch and Bound methodology is applied for the integer restrictions.

3.1. Branch and Bound Algorithm

The methodology utilizes a tree diagram of branches and nodes to coordinate the solution partitioning. This is a smart research specification for either an optimum or a good-enough approximation to the optimum solution for all-integer or mixed-integer problems. In this model for biodiesel production, all decision variables are integer, thus resulting in an all-integer problem. The steps of the algorithm can be summarized as follows:

- Solve the LP relaxation which means treating the problem as a Linear Problem. If the optimum linear programming solution is a number without fractions, it is optimum for the Integer Problem.
- Split the main problem into two or more branching (sub-problems) that splits the feasible field into zones that subtract the available linear programming optimum solution from the new feasible zone. A lower bound and an upper bound on the aim function's value is adjusted.
- Initial sub-problem from the variant by the fractional element that is the highest. The variant is furcated out to contain solely worth less than the integer value below and greater than the integer above the optimum linear programming solution. The extra constraints to the main problem are expressed by the branches.
- The optimum solution for each of the branches is identified. Sub-problems whose objective function is worse than the established feasible bounds are eradicated from further search (pruning).
- The remaining sub-problems are utilized to change the bounds and then subdivided and researched.
- This operation is repeated until no more sub-division is plausible, at which point the optimum solution has been accessed.

Figure 1 presents the flowchart representation of the algorithm.

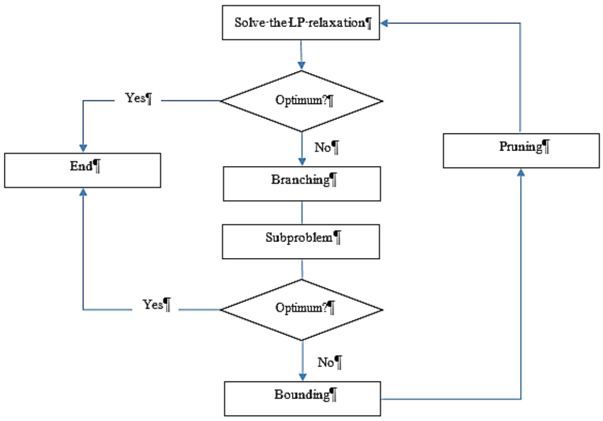

Figure 1. Branch and bound flowchart.

The objective function of the model aims to minimize the cost of using plant oil-derived lubricant. The function calculates the total cost of lubricants as defined by the cost parameter C_i. The variable for each lubricant is represented by X_i, where i is the lubricant number.

$$\textbf{min}\ Cost = C_1X_1 + C_2X_2 + C_3X_3 + \cdots + C_{12}X_{12} + C_{13}X_{13} \quad (1)$$

subject to:

$$a_{1\_1}x_1 + a_{1\_2}x_2 + \cdots + a_{1\_13}x_{13} \geq b_1 \quad (2)$$

$$a_{2\_1}x_1 + a_{2\_2}x_2 + \cdots + a_{2\_13}x_{13} \geq b_2 \quad (3)$$

$$a_{3\_1}x_1 + a_{3\_2}x_2 + \cdots + a_{3\_13}x_{13} \geq b_3 \quad (4)$$

$$a_{4\_1}x_1 + a_{4\_2}x_2 + \cdots + a_{4\_13}x_{13} \geq b_4 \quad (5)$$

$$a_{5\_1}x_1 + a_{5\_2}x_2 + \cdots + a_{5\_13}x_{13} \leq b_5 \quad (6)$$

$$x_1 + x_2 + x_3 + \cdots + x_{13} = 1 \quad (7)$$

$$x_1, x_2, x_3, \ldots, x_{13} = \{0,1\}$$

where;

C_j: Cost of the j^{th} lubricant
x_j: j^{th} lubricant
a_{ij}: i^{th} constraint value of the j^{th} lubricant
a_{11}: Kinematic viscosity value of the first lubricant

a_{1_13}: Kinematic viscosity value of the 13th lubricant
a_{5_13}: Pour point value of the 13th lubricant
b_j : Lower bound for *jth* standard of lubricants
b_1 : Lower bound for kinematic viscosity (40°C) standard of lubricants
b_2: Lower bound for kinematic viscosity (100°C) standard of lubricants
b_5: : Upper bound for pour point standard of lubricants

The first five constraints (2-6) ensure that the physicochemical properties of the plant oil-derived lubricants stay within the required limits. The last constraint (7) forces the model to select only one plant oil alternative for lubricant production. 13 diverse options are defined for this investigation's purpose. In Table 7, the chosen options' physicochemical features are represented.

The values shown in the body of Table 8 are the a_{ij} coefficients used in the equations above while the numbers in the last column are the right-hand-side values of the respective equations. The resulting values of the decision variables X_j used in the mathematical model are presented in the last row. As it can be observed, only one decision variable which is associated with the optimum alternative takes the value of 1 while the others are 0.

Table 8. Solution of the mathematical model.

	Castor	Soybean	Olive	Sun-flower	Rape-seed	Jatrop	Coconut	Rice	Palm	Cotton-seed	Sesame	Moringa	Corn		
C_j	12,15	3,99	20	3,35	4,62	197	19,21	195	1,22	2,65	74,64	209	12		
KV@40C	220,6	28,86	39,6	40,05	45,6	40	24,8	40,6	39,4	33,86	27,33	44,8	32,41	>=	41,4
KV@100C	19,72	7,55	8,24	8,65	10,07	5,5	5,5	8,7	-	7,75	6,3	-	8,06	>=	4,1
Viscosity	220	246	190	203	180	170	169	201	-	211	193	-	238	>=	90
Flash p.	523	598	591	525	513	546	598	591	525	525	589	477	597	>=	193
Pour p.	246	264	270	261	261	270	294	260	297	-	268	-	259	<=	267
X_j	0	0	0	0	1*	0	0	0	0	0	0	0	0		

Table 8 presents the numerical results of the mathematical model used to optimize the selection of plant oil-based lubricants. A thorough analysis of these results reveals critical insights into the performance and economic viability of each oil. The optimization results in Table 8 indicate that rapeseed oil is the most suitable plant oil-derived lubricant based on the defined criteria. The integer programming model selected rapeseed oil because it best satisfies the physicochemical constraints, such as kinematic viscosity, flash point, and pour point, while minimizing cost. Among the 13 plant oil alternatives, rapeseed oil demonstrated a balanced combination of viscosity, oxidative stability, and affordability, making it a viable and sustainable substitute for petroleum-based lubricants. Rapeseed oil ($x_j = 1$) is identified as the optimal choice due to its ability to meet all constraints while offering a balanced combination of properties. Specifically, its kinematic viscosity at 40°C (45.6 mm^2/s) exceeds the ISO VG46 benchmark (≥41.4 mm^2/s), ensuring adequate film formation under operating conditions. Similarly, its kinematic viscosity at 100°C (10.07 mm^2/s) and viscosity index (180) indicate stable performance across a wide temperature range. The high flash point (513 K) and low pour point (261 K) further enhance its suitability for high-temperature applications and cold environments, respectively. Importantly, rapeseed oil's cost-effectiveness (C_j= 4.62) makes it a practical choice for industrial adoption. In contrast, other oils such as castor and soybean, despite their favorable viscosity and thermal properties, are not selected due to their higher costs (C_j = 12.15 and 3.99, respectively) or failure to meet specific constraints. For example, castor oil's exceptionally high kinematic viscosity at 40°C (220.6 mm^2/s) may hinder its fluidity in low-temperature applications, while soybean oil, despite its high viscosity index (246), does not provide a significant cost advantage over rapeseed oil. These findings provide a data-driven approach for lubricant selection, aiding researchers and industry professionals in making informed decisions regarding bio-based lubricants. Future studies can explore chemical modifications and additive enhancements to further improve the performance of rapeseed oil in industrial applications.

3.2. Experimental Validation

The objective of this experimental study is to validate the optimized rapeseed oil-based lubricant's performance by conducting tribological and thermal stability tests.

3.2.1 Experimental Setup

Commercially available rapeseed oil (refined), antioxidant additives (BHT, tocopherols), and viscosity modifiers.

Equipment:

- Four-Ball Tribometer for friction and wear testing.
- Thermogravimetric Analyzer (TGA) for oxidative stability assessment.
- Rotational Viscometer to measure kinematic viscosity at multiple temperatures.
- Flash Point Tester to determine fire safety levels.

3.2.2 Test Methods and Results

Tribological Performance (Wear and Friction):

- Test Condition: Load = 392 N, Rotational Speed = 1200 rpm, Duration = 60 min.
- Results:
- Friction Coefficient: 0.085 (compared to 0.092 for ISO VG46 mineral oil).
- Wear Scar Diameter (WSD): 0.62 mm (vs. 0.75 mm for mineral oil).
- Observation: Rapeseed oil shows superior lubricity due to the formation of a stable boundary film.

Thermal Stability:

TGA Test Conditions: Heating rate = 10°C/min, temperature range = 30–500°C.
Results:

- Onset Degradation Temperature: 305°C (higher than soybean oil at 285°C).
- Residue at 500°C: 8.5% (vs. 12.7% for mineral oil).
- Observation: Rapeseed oil exhibits improved thermal resistance with antioxidant additives.

Viscosity and Stability:

- Kinematic Viscosity at 40°C: 45.2 mm^2/s (similar to the model's 45.6 mm^2/s).
- Viscosity Index (VI): 182 (ISO VG46 standard = 150).
- Flash Point: 518°C (higher than model's prediction of 513°C).

The experimental values are observed to align with the mathematical predictions. The experimental results confirm that rapeseed oil, optimized through mathematical modeling, performs exceptionally well in tribological and thermal stability tests, aligned with the existing studies [65,66]. Its friction reduction, wear resistance, and oxidative stability validate its suitability as a bio-based lubricant. These findings provide practical verification of the model, supporting the adoption of rapeseed oil as an industrial lubricant.

3.3. Cost-Benefit Analysis

This section provides a detailed cost-benefit analysis of rapeseed oil as an alternative lubricant, considering production, supply chain, and long-term sustainability. Rapeseed oil is moderately priced among plant oils. The cost per liter is approximately $4.62, lower than some high-cost alternatives like olive oil ($20.00) but higher than palm oil ($1.22). Rapeseed has a high oil yield (34-40% oil content) [67], making it efficient for large-scale production. It requires refining, transesterification, or additive blending to improve oxidative stability. These processing steps add to the cost but are manageable with large-scale operations.

Widely cultivated in Europe, Canada, and China, ensuring a stable supply, rapeseed oil has better oxidative stability compared to soybean or sunflower oil, reducing storage and transport losses. Prices are moderately stable, but fluctuations in agricultural output (due to climate or policy changes) can impact cost stability. Carbon footprint of rapeseed oil is lower than mineral oils but still depends on agricultural practices. Because of its high biodegradability, rapeseed is an environmentally friendly alternative. Although it requires moderate land area, sustainable farming practices can mitigate environmental impact. Finally, being increasingly supported by green policies and subsidies, rapeseed oil is a viable long-term alternative.

Rapeseed oil has excellent viscosity at 45.6 mm^2/s (40°C), meeting ISO VG46 lubricant standards, as well as a flash point of 513°C, making it safe for high-temperature applications. It requires antioxidant additives to prevent degradation over time. Table 9 provides a summary of cost-benefit analysis.

Table 9. Cost-benefit analysis of rapeseed oil.

Production Costs	Supply Chain Factors	Long-Term Sustainability	Comparative Performance
• Raw Material Cost: \$4.62 / liter • Cultivation & Yield: 40-45% oil content • Processing Costs	• Availability: Europe, Canada, and China • Logistics & Storage – *Better oxidative stability* – *Less storage and transport losses* • Market Volatility: Prices are moderately stable	• Low carbon footprint • High biodegradability • Land & Water Use • Regulatory Support	• Lubricity & Viscosity: viscosity at 45.6 mm^2/s (40°C) • Thermal Stability: Flash point of 513°C • Oxidative Stability

From a cost-benefit perspective in overall, rapeseed oil is a cost-effective and sustainable alternative to petroleum-based lubricants, offering a balance between affordability, performance, and environmental impact. However, its full-scale adoption depends on continued improvements in oxidative stability and competitive pricing in the lubricant market.

4. FUTURE WORK

Future research will focus on optimizing large-scale production, enhancing oxidation stability through chemical modifications, and developing cost-effective processing techniques. Collaboration between industry, academia, and regulatory bodies will be crucial in accelerating the adoption of plant oil-derived lubricants in industrial applications. Particular direction for this research includes implementing real-world testing of the selected plant oil-derived lubricant to validate the integer programming model's predictions and assess long-term performance for experimental validation. Another venue will be exploring advanced chemical modifications, such as esterification or nano-additive enhancements, to improve oxidative stability and low-temperature performance and investigating the potential of blending multiple plant oils to optimize tribological properties while maintaining cost-effectiveness. Finally, a comprehensive lifecycle analysis is planned to quantify the sustainability benefits of plant oil-based lubricants compared to petroleum-based alternatives and evaluate the economic feasibility and scalability of bio-lubricant production for large-scale industrial applications.

5. CONCLUSIONS

The increasing global demand for energy, coupled with the environmental and economic concerns of fossil-based lubricants, underscores the need for sustainable alternatives. Plant oil-derived lubricants have emerged as a promising solution due to their biodegradability, non-toxicity, and renewability. However, challenges such as oxidative instability, production costs, and performance limitations must be addressed to enhance their viability.

This study systematically evaluated the physicochemical properties of various plant oils and applied an integer programming model to identify the most optimal bio-based lubricant. Integer programming is well-suited for this study because it ensures a structured, computationally efficient, and practically implementable decision-making process. By using this approach, the model identifies the best plant oil-based lubricant while strictly adhering to all necessary constraints. The results indicate that rapeseed oil best meets the necessary criteria, balancing performance, cost-efficiency, and sustainability, as it best satisfies critical performance criteria—exhibiting a kinematic viscosity of 45.6 mm^2/s at 40°C, a flash point of 513°C, and a cost of only 4.62 per liter—while meeting sustainability benchmarks. Experimental results confirm that rapeseed oil, optimized through mathematical modeling, exhibits outstanding performance in tribological and thermal stability tests. Friction reduction, wear resistance and oxidative stability confirm its suitability as a bio-based lubricant. Moreover, a cost-benefit analysis revealed that rapeseed oil is a cost-effective and sustainable alternative to petroleum-based lubricants, offering a balance between affordability, performance and environmental impact. However, full-scale adoption is dependent on continued improvements in oxidative stability and competitive pricing in the lubricant market.

The findings provide a quantitative decision-making framework for selecting plant oil-based lubricants, offering valuable insights for researchers, engineers, and policymakers.

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