DOI: 10.24874/ti.1233.12.21.03

Tribology in Industry

www.tribology.rs

RESEARCH

# A Study on the Relationship Between Contact Pressures in a Grade2 TDN–UHMWPE Hip Implant: Young's Modulus and Poisson's Ratio

M. Chernets[a,*], Yu. Chernets[a], J. Zubrzycki[b]

[a]*Aerospace Faculty, National Aviation University, Lubomyr Huzar 1, 03058 Kyiv, Ukraine,*
[b]*Lublin University of Technology, Nadbystrzycka str. 36, 20-618 Lublin, Poland.*

Keywords:

*Hip joint implant*
*Calculation method*
*Maximum contact pressures*
*Tribological pair: Grade2TDN – UHMWPE*

* Corresponding author:

*Myron Chernets*
*E-mail: myron.czerniec@gmail.com*

*Received: 21 December 2021*
*Revised: 7 February 2021*
*Accepted: 2 March 2022*

A B S T R A C T

*Statistical data show that over 90% hip implants have the acetabular cup made of ultra-high molecular weight polyethylene (UHMWPE), while the femoral head is made of stainless steel, CoCrMo alloy, titanium and ceramics. The values of Young's modulus for stainless steel, CoCrMo alloy and titanium differ from that for UHMWPE by 100–200 times, while Young's modulus of ceramics differs from the Young's modulus value of UHMWPE by up to 400 times. Young's modulus of UHMWPE can also differ by up to 3 times, which considerably affects contact parameters. Therefore, the problem of assessing the impact of the elastic characteristics of UHMVE on the contact pressure in the prostheses of the hip joint is relevant.*

*A new calculation method is used to determine the effect of Young's modulus and Poisson's ratio (UHMWPE) on maximum contact pressures in a hip implant. This study undertakes an analysis of the relationship between elastic properties of UHMWPE and contact pressures, considering a Grade2 TDN (thermo-diffusion nitrided titanium) femoral head diameter and radial clearance. Quantitative and qualitative relationships are established between contact pressures and Young's modulus and Poisson's ratio for the UHMWPE acetabular cup. The proposed calculation method can be used at the stage of hip implant selection to estimate contact pressures depending on the implant load (patient's weight and muscular force) and geometry (femoral head diameter and radial clearance in the implant). Contact pressures also have a significant impact on the wear rate of an UHMWPE acetabular cup, as well as on hip implant service life.*





## 1. INTRODUCTION

Statistic show that in over 90% hip implants the acetabular cup is made of ultra-high molecular weight polyethylene (UHMWPE) [1,2]. On the other hand, the femoral head is made of stainless steel, CoCrMo alloy, titanium and alumina ceramics, these materials having considerably higher values of Young's modulus than UHMWPE. According to the literature, the difference between their Young's modulus values can be as high as 100-200 times, and for aluminum – as high as 400 times. As far as Poisson's ratio is concerned, the difference is relatively small and does not exceed 1.5 times.

Few studies have focused on the relationship between Young's modulus of UHMWPE and maximum contact pressures. This problem was partly investigated with respect to contact pressures and contact area in [3] and [9] by Finite Element Method (FEM) and experimental tests. In addition, the scientific literature lacks comprehensive studies investigating the effect of Young's modulus $E_1$ and Poisson's ratio $\nu_1$ on contact parameters, i.e. maximum contact pressures, angle of contact, radius and area of contact.

A review of literature [1,4,5–8,10–15 et al.] on FEM analysis of hip implants provides different data about the above-mentioned contact parameters for different values of Young's modulus and Poisson's ratio (Table 1). It should be emphasized that UHMWPE is characterized by reduced thermal resistance, therefore its Young's modulus can be reduced by 15–20% [16] due to contact with human body temperature, while at the same time its Poisson's ratio will increase by 1-1.5%.

**Table 1.** Elastic properties of UHMWPE.

| Authors | [4] | [5] | [5] | [6] | [1] | [7] | [8] | [9] | [12] | [13] | [14] | [16] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $E_1$, MPa | 1000 | 840 | 680 | 1400 | 880 | 500 | 850 | 1000 | 2200 | 1000 | 1400 | 750 |
| $\nu_1$ | 0.4 | 0.44 | 0.425 | 0.46 | 0.4 | 0.4 | 0.4 | 0.4 | 0.33 | 0.45 | 0.4 | 0.46 |

In light of the above, the determination of a relationship between elastic properties of UHMWPE and contact pressures in hip implants is of vital significance not only for scientific reasons, but also for utilitarian purposes in terms of loads acting on this biomechanical system. This study uses a calculation method developed for hip implant analysis [17] to investigate the effect of Young's modulus and Poisson's ratio of UHMWPE on contact pressures in the implant and to establish relationships there between.

## 2. RESEARCH METHOD

At present, there is no analytical solution on the almost conformal contact of the sphere with the spherical cavity. In a recent paper [18] using the model of the Winkler foundation elastic deformation, an analytical solution of such a problem was obtained to determine the pressure distribution. The following is a calculation method in which a ball hip implant (3D system) (Fig. 1) is modeled as a cylindrical joint (3D system) (Fig. 1) with a model (effective) radius.

The 3$D$ system is then reduced to a 2$D$ system by reducing the total compressive load $N$ (Fig. 2) acting on the femoral head in the implant to a unit diameter of the head, i.e. $N' = N / D_2$. Accordingly, in the modelled joint, $R_1 \approx R_2 = R$ ($R_1$, $R_2$ - sleeve radius 1 and disk radius 2). Both the hip implant and the model are described by a radial clearance of $\varepsilon = R_1 - R_2 \geq 0 \langle\langle R$. The load $N'$ induces contact between the two elements in a zone described by an angle $2\alpha_0$ and contact pressures $p(\alpha)$.

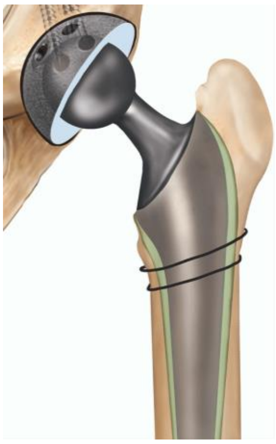


**Fig. 1.** Hip implant.

Various methods for solving plane contact problems on the internal contact of cylindrical bodies with equal or close radii have been developed in [19-24, etc.]. In 1954 [21,22] was established an equation for determining contact stresses, using the equality condition of adjacent bodies curvatures in the contact area. The collocation method was used to solve it. The solution of several such a class of problems using this approach is considered in the monograph [24].

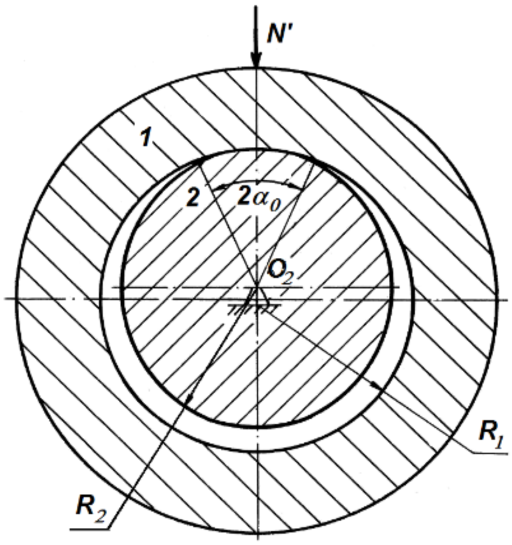



**Fig. 2.** Schematic illustrating calculations for a cylindrical joint.

Later, such a flat contact problem on the distribution of contact stresses was investigated [25] using a very similar equation [21]. Using the results of [25], in [26,27] were obtained solutions of plane problems for bodies with equal and different elastic characteristics. Previously, such problems were also studied in [24].

In [25-30] the generalized methods of calculation of the specified cylindrical joints are developed, where a small facet (ellipticity, ovality, tri- and quadrilateral) of connected bodies contours was taken into account. According to these methods, studies of the hip arthroplasty as a model 2D cylindrical hinge with ovality of the cylinder (head) were performed [31].

In the model hinge, bodies with circular contours are considered below as in [17]. Solving the contact problem is solved by determining maximum contact pressures $p(0)$, their distribution $p(\alpha)$ and contact angle $2\alpha_0$. For this two-dimensional contact mechanics problem the formula describing contact pressures $p(\alpha)$ has the form [25,28]

$$c_1 \int_{-\alpha_0}^{\alpha_0} \cot\frac{\alpha-\theta}{2} p'(\theta)d\theta = c_2 p(\alpha) + c_3 \int_{-\alpha_0}^{\alpha_0} p(\alpha)d\alpha$$

$$+ \; c_4 \cos\alpha \int_{-\alpha_0}^{\alpha_0} p(\alpha)\cos\alpha d\alpha + \frac{\varepsilon}{R^2}, \qquad (1)$$

where $p'(\theta) = dp/d\theta$; $\alpha$ is a polar angle; $0 \le \alpha \le \theta$, $-\alpha_0 \le \theta \le \alpha_0$;

$$c_1 = \frac{1}{8\pi R}\left(\frac{1+\kappa_1}{G_1} + \frac{1+\kappa_2}{G_2}\right);\; c_2 = \frac{1}{4R}\left(\frac{1-\kappa_1}{G_1} - \frac{1-\kappa_2}{G_2}\right);$$

$$c_3 = \frac{1+\kappa_1}{8\pi G_1 R};\; c_4 = \frac{1}{2\pi R}\left(\frac{\kappa_1}{G_1} + \frac{1}{G_2}\right);$$ are

generalized characteristics of elasticity; $G_1, G_2$ are shear modulus values of the materials; $\nu_1, \nu_2$ are Poisson's ratio values of the materials; $\kappa = 3 - 4\nu$ corresponds to the plane state of strain.

The calculations for the modelled 2D system (cylindrical joint), which substitutes a 3D system described by the radii $R_1$, $R_2$ of hip implant elements, are performed using an effective radius of $R_* = 0.5\sqrt{R_1 R_2} = 0.5\sqrt{(R_2+\varepsilon)R_2}$.

An approximate solution of Equation (1) is produced with the use of collocation method [16,19]. For two points of a collocation of $\alpha = \pm 0.5\alpha_0$, a function of contact pressures $p(\alpha)$ has the following form [25,28]:

$$p(\alpha) \approx E_0 \varepsilon \sqrt{\tan^2\frac{\alpha_0}{2} - \tan^2\frac{\alpha}{2}}, \qquad (2)$$

where the simplified ratio for the collocation coefficient is as follows:

$$E_0 = (e/R_*)\cos^2(\alpha_0/4), \qquad (3)$$

where $e = 4E_1E_2/Z$,

$$Z = (1+\kappa_1)(1+\nu_1)E_2 + (1+\kappa_2)(1+\nu_2)E_1,$$

$E = 2G(1+\nu)$ is Young's modulus of the material.

The maximum contact pressures $p(0)$ occur when $\alpha = 0$. Then

$$p(0) \approx E_0 \varepsilon \tan\frac{\alpha_0}{2}. \qquad (4)$$

An unknown half-angle of contact, $\alpha_0$, is determined from the condition of equilibrium of forces acting on disk 2

$$N' = R_* \int_{-\alpha_0}^{\alpha_0} p(\alpha)\cos\alpha d\alpha = 4\pi R_* E_0 \varepsilon \sin^2(\alpha_0/4) . \quad (5)$$

Considering the formula for Young's modulus $E_0$, Equation (5) taking into account (3) has the following form

$$N' = \pi e \varepsilon \sin^2(\alpha_0/2). \quad (6)$$

Hence, the half-angle of contact $2\alpha_0$

$$\alpha_0 = 2arc\sin\sqrt{N'/\pi e\varepsilon} . \quad (7)$$

The following complete formula can be used for the coefficient $E_0$ to increase the accuracy of the approximate solution of the problem by the given method:

$$E_0 = \frac{\varepsilon e}{R}\left[\begin{array}{l}\cos^{-2}\frac{\alpha_0}{4} - e_1\sqrt{\tan^2\frac{\alpha_0}{2} - \tan^2\frac{\alpha_0}{4}} - \\ 0.5\sin^2\frac{\alpha_0}{4}\left(e_2\cos^{-1}\frac{\alpha_0}{2} + 2e_3\cos\frac{\alpha_0}{2}\right)\end{array}\right]^{-1}, \quad (8)$$

where

$$e_1 = \frac{2}{Z}\left[(1-\kappa_1)(1+\nu_1)E_2 - (1-\kappa_2)(1+\nu_2)E_1\right],$$

$$e_2 = \frac{2}{Z}(1+\kappa_1)(1+\nu_1)E_2,$$

$$e_3 = \frac{4}{Z}\left[\kappa_1(1+\nu_1)E_2 + (1+\nu_2)E_1\right].$$

When using the simplified formula (3) to find $E_0$, the pressure $p(0)$ at the contact angle $2\alpha_0$ =100° is lower by up to 4% compared to the exact solution [24]. When using formula (8) to find $E_0$, the pressure $p(0)$ at the contact angle $2\alpha_0$ =100° will be correspondingly lower up to 2.8%. If we use a solution with three collocation points ($\alpha = 0,\ \alpha = \pm 0.65\alpha_0$), the accuracy of the solution will increase. In particular, at a contact angle $2\alpha_0$ =100° the deviation of $p(0)$ will be up to 0.65%. However, then the formulas for determining the two collocation coefficients for points $\alpha = \pm 0.65\alpha_0$ are cumbersome [24, 25, 28].

When solving the above contact problem (Fig. 2), the thickness of the ring (acetabular cup) $h$ = $R_0$- $R_1$ is important, where $R_0$ is its outer diameter. In the literature, MES solutions for hip arthroplasty are known, where $h$ = 4 - 14 mm ([1] - 4-12 mm, [7] - 8.7 mm, [8] - 6 mm, [9] - 9 mm, [11] - 9.42 mm, [12] - 7 mm, [13] - 14 mm, [15] - 14, 12, 16 mm, [18] - 4, 6, 8, 14 mm, etc.). Also, in [5] it was noted that already at h = 10 mm the influence of thickness on the maximum contact pressures is small. Therefore, it should be expected that at $h \approx 8$ mm the contact stresses will not be significantly different [3,18] than those obtained from the solution by the above method.

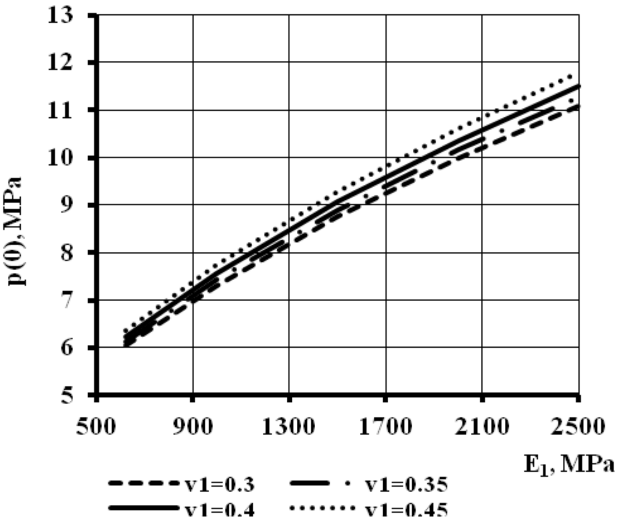
## 3. RESULTS AND DISCUSSION

Numerical calculations of contact parameters for the analyzed metal-on-polymer (MoP) type hip implant were performed using the following data:

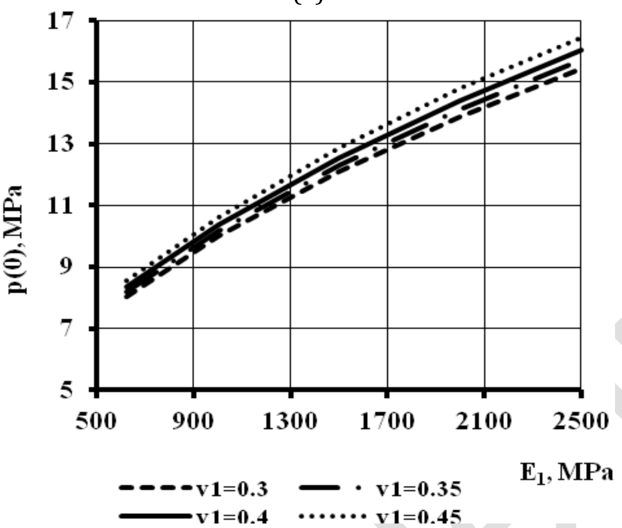
- femoral head diameters: $D_2$ = 28, 38, 48 mm;
- total compressive force $N$ (load) on the femoral head at walking [32,33] for the patient's weight of $K$ = 700 N: $N_{\max}$ = 2900 N, $N_s$ = 1900 N;
- radial clearance: $\varepsilon$ = 0.1, 0.2 mm;
- material of femoral head 2; thermo-diffusion nitrided titanium (GRADE2 TDN) described by Young's modulus $E_2$ = 112000 MPa and Poisson's ratio $\nu_2$ = 0.32;
- material of acetabular cup 1: UHMWPE described by Young's modulus $E_1$ = 625, 1000, 1500, 2000, 2500 MPa, and Poisson's ratio $\nu_1$ = 0.3, 0.35, 0.4, 0.45.

$E_1$ = 625 MPa [17] (37°C) as 80% of $E_1$ = 750 MPa (23° C) [15].

Results obtained for a mean load of $N_s$ = 1900 N are given in Figs. 3–8. Results of $p(0)$ obtained for a femoral head diameter of $D_2$ = 28 mm are given in Figs. 3, 4.




(a)




(b)

**Fig. 3.** Elastic properties of UHMWPE versus contact pressures: (a) $\varepsilon$ = 0.1 mm, (b) $\varepsilon$ = 0.2 mm.

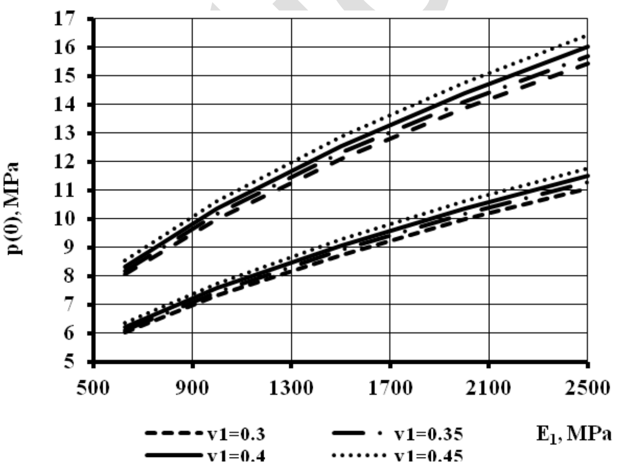
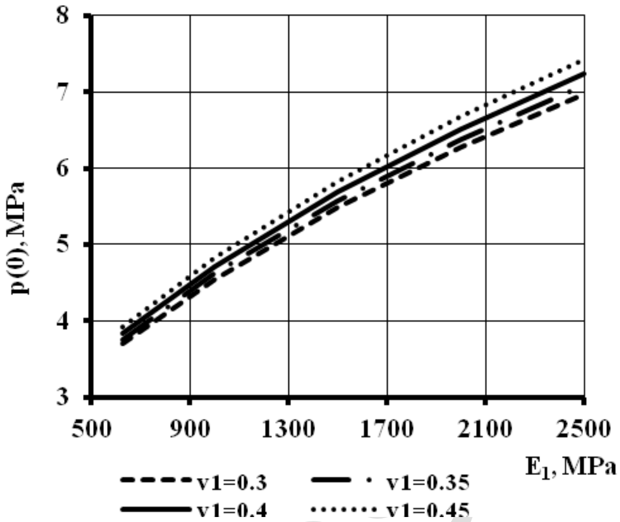
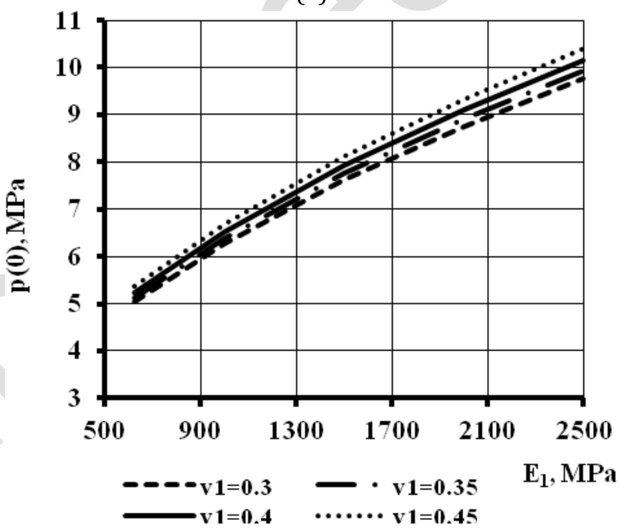



**Fig. 4.** Influence of radial clearance on contact pressures of $D_2$ = 28 mm.

Results obtained for a femoral head diameter of $D_2$ = 38 mm are given in Figs. 5, 6.




(a)




(b)

**Fig. 5.** Elastic properties of UHMWPE versus contact pressures: (a) $\varepsilon$ = 0.1 mm, b) $\varepsilon$ = 0.2 mm.

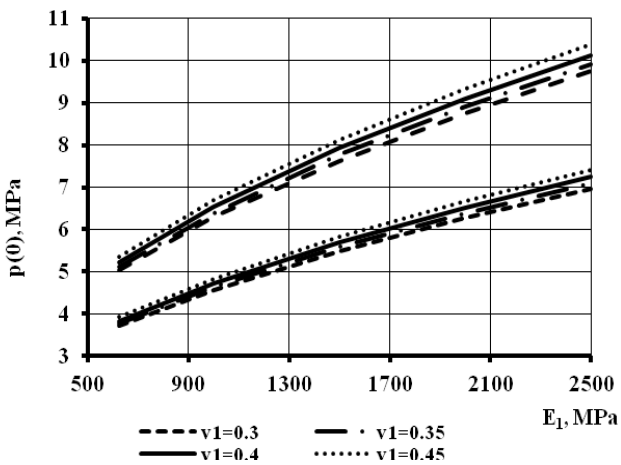



**Fig. 6.** Influence of radial clearance on contact pressures of $D_2$ = 38 mm.

Figs. 7, 8 show the results obtained for a femoral head diameter of $D_2$ = 48 mm.

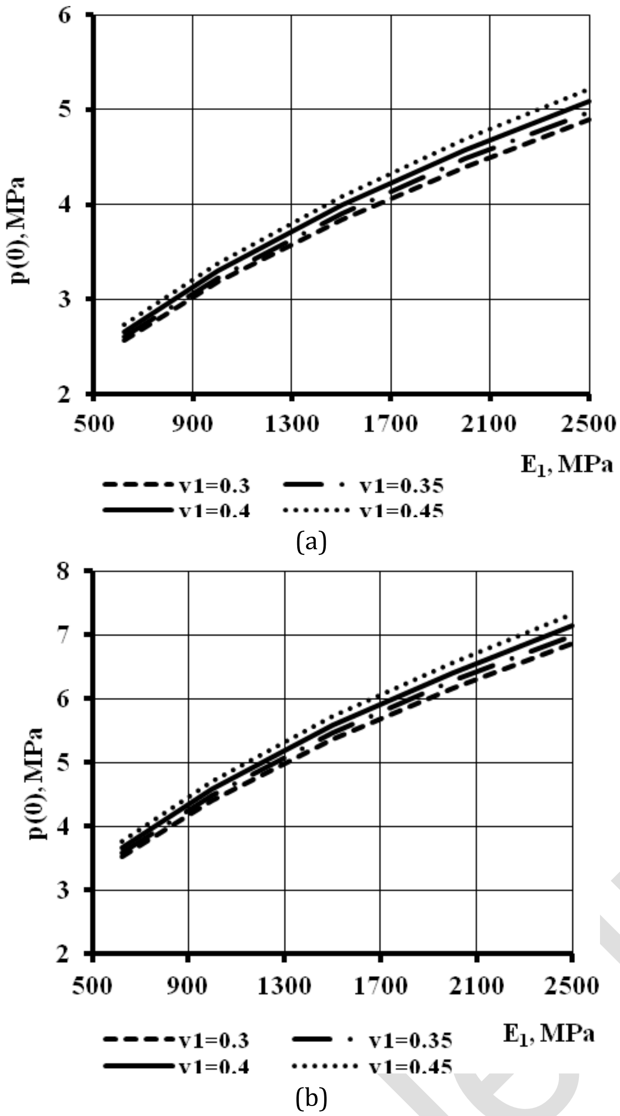



**Fig. 7.** Elastic properties of UHMWPE versus contact pressures: (a) $\varepsilon$ = 0.1 mm, (b) $\varepsilon$ = 0.2 mm.

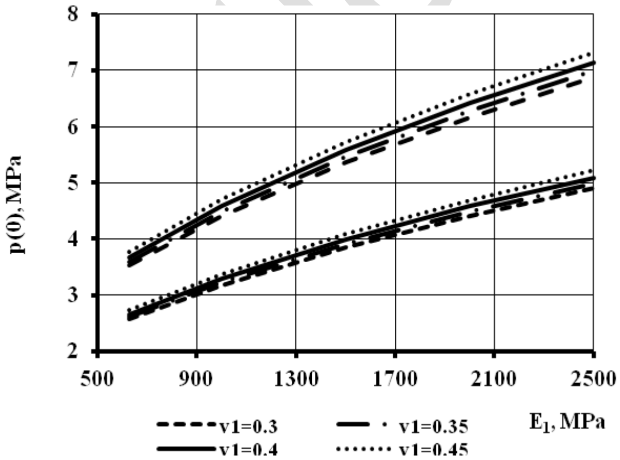



**Fig. 8.** Influence of radial clearance on contact pressures of $D_2$ = 48 mm.

The numerical results demonstrate that the elastic properties of the acetabular cup material (UHMWPE) have a significant impact on contact pressures. It can be observed that the qualitative relationships between contact pressures given in Figs. 3–8 and different values of both Young's modulus and Poisson's ratio of UHMWPE are practically the same. An increase in Young's modulus of UHMWPE (its rigidity) causes a considerable nonlinear increase in contact pressures depending on the value of femoral head diameter and radial clearance in the system.

Regarding the quantitative relationship between the maximum contact pressures $p(0)$ and the two tested elastic characteristics, the following relationships were established:

a) Young's modulus $E_1$ of UHMWPE

- a four-fold increase in Young's modulus causes a significant increase in $p(0)$, namely:

  by 1.85, 1.92 times when $D_2$ = 28 mm and $\varepsilon = 0.1$, 0.2 mm;

  by 1.88, 1.94 times when $D_2$ = 38 mm and $\varepsilon = 0.1$, 0.2 mm;

  by 1.91, 1.95 times when $D_2$ = 48 mm and $\varepsilon = 0.1$, 0.2 mm;

  for lower values of Poisson's ratio and radial clearance the increase is smaller;

- when $E_1$= const and $\nu_1$ ranges 0.3 – 0.45, the increase in pressures $p(0)$ is insignificant, namely:

  $D_2$ = 28 mm: $\varepsilon = 0.1$ mm by 1.051-1.063 times; $\varepsilon = 0.2$ mm by 1.061-1.065 times;

  $D_2$ = 38 mm: $\varepsilon = 0.1$ mm by 1.057-1.065 times; $\varepsilon = 0.2$ mm by 1.063-1.066 times;

  $D_2$ = 48 mm: $\varepsilon = 0.1$ mm by 1.062-1.065 times; $\varepsilon = 0.2$ mm by 1.065-1.066 times;

b) Poisson's ratio $\nu_1$ of UHMWPE

- increasing the value of Poisson's ratio by 1.5 times causes a small increase in $p(0)$ ranging 1.066 times depending on the value of $E_1$, $D_2$ and $\varepsilon$ ;
- when $\nu_1$= const and $E_1$ ranges 625–2500 MPa, the increase in pressures $p(0)$ is as high as 1.95 times depending on the value of $D_2$ and $\varepsilon$ .

For the acetabular cup made of UHMWPE with reduced rigidity ($E_1$ = 625 MPa, $\nu_1$ = 0.45) and increased rigidity ($E_1$ = 2500 MPa, $\nu_1$ = 0.3), one can observe significant differences between maximum contact pressures:

$D_2$ = 28 mm: 1.75 times - $\varepsilon = 0.1$ mm, 1.81 times - $\varepsilon = 0.2$ mm;

$D_2$ = 38 mm: 1.78 times - $\varepsilon = 0.1$ mm, 1.82 times - $\varepsilon = 0.2$ mm;

$D_2$ = 48 mm: 1.79 times - $\varepsilon = 0.1$ mm, 1.83 times - $\varepsilon = 0.2$ mm.

Apart from the elastic properties of UHMWPE, contact pressures are also affected by the geometric parameters of the system, i.e. the femoral head diameter $D_2$ and the radial clearance ε. An increase in the femoral head diameter $D_2$ by 1.714 times leads to a decrease in $p(0)$ by approx. 2.26 times depending on the value of $E_1$ and $\nu_1$. A two-fold increase in the radial clearance $\varepsilon$ leads to increase in the contact pressures $p(0)$ by 1.33–1.37 times (when $E_1$ = 625 MPa and $\nu_1$ = 0.3–0.45) and by 1.39–1.4 times (when $E_1$ = 2500 MPa and $\nu_1$ = 0.3–0.45). With a larger value of the radial clearance ($\varepsilon$ = 0.2 mm), there is a more intense increase in the contact pressures $p(0)$ than at its smaller value ($\varepsilon$ = 0.1 mm), as shown in Fig. 4, 6, 8.

In the studied hip arthroplasty, the module $E_2$ of the acetabular cup 2 made of UHMWPE is much smaller than the modules $E_1$ of the metal head 1 and $E$ of the metal shell. As the cup thickness h decreases, the contact angle $2\alpha_0$ decreases and the contact pressures $p(0)$ increase. To reduce the pressure, choose UHMWPE with a lower modulus (Fig. 3, 5, 7). Also, the reduction of pressure is achieved by reducing the radial clearence in the endoprosthesis (Fig. 4, 6, 8).

Using the above method and data reported in [1, 8, 10, 12, 14] it was possible to determine pressures $p(0)$ and then compare them with pressures $p_{max}$ (Tab. 2).

Analysis of the results of the maximum contact pressures comparison given in table 2 indicates their certain differentiated correlation. There is no clear relationship between the pressure difference in these solutions with the thickness of the cup.

**Table 2.** Data about maximum contact pressures .

| Authors | $p(0)/p_{max}$, MPa | Prosthesis parameters |
|---|---|---|
| [1] | 8.77 / 10.65 (0.82 times)<br><br>11.81 / 13.75 (0.86 times) | $N$ = 2650 N; $D_2$ = 28 mm; $\varepsilon$ = 0.1 mm; Ti – UMHWPE<br>$N$ = 2650 N; $D_2$ = 28 mm; $\varepsilon$ = 0.2 mm; Ti – UMHWPE<br>$E_1$ = 880 MPa; $\nu_1$ = 0.4: $h$ =10 mm |
| [8] | 15.24 / 17.1 (0.89 times) | $N$ = 2500 N; $D_2$ = 22,225 mm; $\varepsilon$ = 0.1825 mm; Stainless – UMHWPE<br>$E_1$ = 850 MPa; $\nu_1$ = 0.4<br>$h$ =8,7 mm |
| [10] | 9.99 / 10.75 (0.93 times) | $N$ = 2500 N; $D_2$ = 36 mm; $\varepsilon$ = 0.3 mm; Ti – UMHWPE<br>$E_1$ = 1000 MPa; $\nu_1$ = 0.4; $h$ =9 mm |
| [12] | 8.84 / 8.5 (1.04 times)<br><br>9.07 / 8.9 (1.02 times)<br>10.2 /10.2 (1.0 times) | $N$ = 1900 N; $D_2$ = 32 mm; $\varepsilon$ = 0.098 mm; CoCrMo – UMHWPE<br>$N$ = 2000 N<br>$N$ = 2500 N<br>$E_1$ = 2200 MPa; $\nu_1$ = 0.33; $h$ =9,423 mm |
| [14] | 16.7 / 21.1 (0.79 times) | $N$ = 2000 N; $D_2$ = 22 mm; $\varepsilon$ = 0.1 mm; CoCrMo – UMHWPE<br>$E_1$ = 1400 MPa; $\nu_1$ = 0.4; $h$ =14 mm |
| [15] | 7.86 / 6.68 (0.85 times) | $N$ = 2000 N; $D_2$ = 28 mm; $\varepsilon$ = 0.08 mm; Stainless – UMHWPE<br>$E_1$ = 500 MPa; $\nu_1$ = 0.4; $h$ =12 mm |

Note: $p(0)$ - maximum contact pressures according to the proposed method; $p_{max}$ - maximum contact pressures

Given the results presented by the developed method, it is possible to very easily, efficiently and with sufficient accuracy for practical purposes to assess the maximum contact pressures in the MoP hip arthroplasty. The accuracy of the pressure estimate compared to the presented method can be improved according to the methods described in paragraph 2. Numerical solution is provided by simple means, in particular in Excel.

## 4. CONCLUSION

The proposed calculation method made it possible to effectively estimate maximum contact pressures in MoP type hip implants, considering the effect of UMHWPE's elastic characteristics $E_1$ and $\nu_1$, femoral head diameter and radial clearance. As a result, qualitative and quantitative relationships between these parameters have been established.

1. An increase in the values of the elastic characteristics of the acetabular cup material (UMHWPE) leads by approx. two times increase in contact pressures. On the other hand, increasing only the Poisson's ratio value in the tested range leads to increased contact pressures by up to 1.065 times. Therefore, it can be concluded that the Young modulus of UMHWPE plays a key role in contact pressure reduction.

2. A increase in the Grade2 TDN femoral head diameter leads to reduction in contact pressures. A two-fold increase in radial clearance of the system leads to a 1.4 times increase in contact pressures.

3. It has been found that for the acetabular cup made of UMHWPE with lower values of its elastic characteristics, particularly Young's modulus, contact pressures and wear rate of the implant are significantly reduced.

## REFERENCES


[1] V. Grushko, S.E. Sheykin, I. Rostotsckiy, *Contact pressure in hip endoprosthesis swivel joints*, Journal of Friction and Wear, vol. 33, pp. 124–129, 2012, doi: 10.3103/S1068366612020055

[2] A. Poliakov, V. Pakhaliuk, V. Popov, *Current Trends in Improving of Artificial Joints Design and Technologies for Their Arthroplasty*, Frontiers in Mechanical Engineering, Online articles, 2020, doi: 10.3389/fmech.2020.00004

[3] D.L. Bartel, A.H. Burstein, M.D. Toda, D.L. Edwards, *The effect of conformity and plastic thickness on contact stress in metal-backed plastic implants*, Journal of Biomechanical Engineering, vol. 107, iss. 3, pp. 193–199, 1985, doi: 10.1115/1.3138543

[4] Z.M. Jin, S.M. Heng, H.W. Ng, D.D. Auger, *An axisymmetric contact model of ultra high molecular weight polyethylene cups against metallic femoral heads for artificial hip joints replacements,* Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine, vol. 213, iss. 4, pp. 317–327, 1999, doi: 10.1243/0954411991535158

[5] G.R. Plank, D.M. Estok, O.K. Muratoglu, D.O. O'Connor, B.R. Burroughs, W.H. Harris, *Contact stress assessment of conventional and highly crosslinked ultra high molecular weight polyethylene acetabular liners with finite element analysis and pressure sensitive film,* Journal of Biomedical Materials Research Part B: Applied Biomaterials, vol. 80, iss. 1, pp. 1–10, 2007, doi: 10.1002/jbm.b.30560

[6] S. Barreto, J. Foldago, P.R. Fernandes, J. Monteiro, *The influence of the pelvic bone on the computational results of the acetabular component of a total hip prosthesis,* Journal of Biomechanical Engineering, vol. 132, iss. 5, pp. 1–4, 2010, doi: 10.1115/1.4001031

[7] F. Liu, J. Fisher, Z. Jin, *Computational modeling of polyethylene wear and creep in total hip joints replacements: Effect of the bearing clearance and diameter,* Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology, vol. 226, iss. 6, pp. 552–563, 2012,doi: 10.1177/1350650112441908

[8] X. Hua, B.M. Wroblewski, Z. Jin, L. Wang, *The effect of cup inclination and wear on the contact mechanics and cement fixation for ultra high molecular polyethylene total hip replacements,* Medical Engineering and Physics, vol. 34, iss. 3, pp. 318–325, 2012, doi: 10.1016/j.medengphy.2011.07.026

[9] X. Hua, L. Wang, M. Al-Hajjar, Z. Jin, R.K Wilcox, J. Fisher, *Experimental validation of finite element modelling of a modular metal-on-polyethylene total hip replacement,* Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine, vol. 228, iss. 7, pp. 682–692, 2014, doi: 10.1177/0954411914541830

[10] X. Hua, J. Li, L. Wang, Z. Jin, R. Wilcox, J. Fisher, *Contact mechanics of modular metal-on-polyethylene total hip replacement under adverse edge loading conditions,* Journal of Biomechanics, vol. 47, iss. 13, pp. 3303–3309, 2014, doi: 10.1016/j.jbiomech.2014.08.015

[11] L. Mattei, F. Di Puccio, E. Ciulli, *A comparative study of wear laws for soft-on-hard hip implants using a mathematical wear model,* Tribology International, vol. 63, pp. 66–77, 2013, doi: 10.1016/j.triboint.2012.03.002

[12] S. Shankar, L. Prakash, M. Kalayarasan, *Finite element analysis of different contact bearing couples for human hip prothesis,* International Journal of Biomedical Engineering and Technology, vol. 11, no. 1, pp. 66–80, 2013, http://dx.doi.org/10.1504/IJBET.2013.053712

[13] Y. Gao, Z. Jin, L. Wang, M. Wang, *Finite element analysis of sliding distance and contact mechanics of hip implant under dynamic walking conditions,* Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine, vol. 229, iss. 6, pp. 1–6, 2015, doi: 10.1177/0954411915585380

[14] M.S. Uddin, *Contact of dual mobility implants: effects of cup wear and inclination,* Computer Methods in Biomechanics and Biomedical Engineering, vol. 18, iss. 15, 1611–1621, 2015, doi: 10.1080/10255842.2014.936856

[15] E. Askari E., M.S. Andersen, *A closed-form formulation for the conformal articulation of metal-on-polyethylene hip prostheses: contact mechanics and sliding distance*, Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine, vol. 232, iss. 12, pp. 1196–1208, 2018, doi: /10.1177/0954411918810044

[16] W. Wielieba, *Maintenance-free plain bearings made of thermoplastic polymers*, Wroclaw: Wroclaw University of Technology, 2013. (in Polish)

[17] M. Czerniec, J. Zubrzycki, *Study of Contact Pressures in Total Hip Replacement,* Advances in Science and Technology Research Journal, vol. 15, iss. 2, pp. 176–183, 2021, doi: 10.12913/22998624/136178

[18] M. Heß, F. Forsbach, *An Analytical Model for Almost Conformal Spherical Contact Problems: Application to Total Hip Arthroplasty with UHMWPE Line,* Applied Sciences, vol. 11, iss. 23, pp. 1–14, 2021, doi: /doi.org/10.3390/app112311170

[19] I.Ya. Steierman, *Contact problem of the theory of elasticity,* Moskow - Leningrad: Gostekhizdat, 1949. (in Russian)

[20] M.P. Sheremetyev, *Elastic balance of an endless plate with an absolutely rigid or elastic washer inserted*, PMM, vol.16, no.4, pp. 437- 448, 1952, (in Russian)

[21] V.V. Panasyuk, *Contact problem for a circular hole,* Questions of Mechanical Engineering and Strength in Mechanical Engineering, vol. 3, no. 2, pp. 59–79, 1954. (in Russian)

[22] V.V. Panasyuk, *Stamp pressure at the boundary of the circular hole*, Reports of the Academy of Sciences of the USSR, no.1, pp. 37–40, 1954. (in Russian)

[23] B.L. Romalis, *Pressure distribution in the internal contact of elastic circular cylinders*, Journal of Mechanical Engineering, no. 1, pp. 13–15, 1958. (in Russian)

[24] V.V. Panasyuk, M.I. Teplyi, *Some contact problems of the theory of elasticity*, Kyiv: Naukova Dumka, 1975. (in Ukrainian)

[25] A.E. Andreikiv, M.V. Chernets, *Evaluation of the Contact Interaction of Rubbing Machine Elements*; Naukova Dumka: Kiev, Ukraine, 1991. (in Russian)

[26] M.V. Chernets, O.E. Andreikiv, N.M. Liebiedieva, V.B. Zhydyk, *A model for evaluation of wear and durability of plain bearing with small non-circularity of its contours*, Materials Science, vol. 45, pp. 279–290, 2009, doi: 10.1007/s11003-009-9176-5

[27] M.V. Chernets, *Contact problem for a cylindrical joint with technological faceting of the contours of its parts*, Materials Science, vol. 45, pp. 859–868, 2009, doi: 10.1007/s11003-010-9252-x

[28] M.V. Chernets, *Tribo-contact tasks for cylindrical joints with technological non-circularity,* Lublin: Lublin University of Technology, 2013. (in Ukrainian)

[29] M. Chernets, *Evaluation of contact strength and durability of plain bearings with different of shaft lobing,* Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology, vol. 229, iss. 12, pp. 1444–1454, 2015, doi: 10.1177/1350650115580629

[30] M. Chernets, Ju. Chernets, *Generalized method for calculating the durability of sliding bearings with technological out-of-roundness of details*, Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology, vol. 229, iss. 2, pp. 216–226, 2015, doi: 10.1177/1350650114554242

[31] M. Chernets, *Calculation Method for Assessing Contact Parameters in the Hip Prosthesis Made of Thermo-diffusion Nitrided Grade 2 / Ultra-high Molecular Weight Polyethylene*, Journal of Biomedical Engineering and Technology, vol. 8, iss. 1, pp. 6–13, 2020, doi: 10.12691/jbet-8-1-2

[32] F. Pauwers, *Biomechanics of the locomotor apparatus,* Springer, 1980, doi: 10.1007/978-3-642-67138-8

[33] B.I. Kupchinov, S.F. Ermakov, E.D. Belonenko, *Biotribology of synovial joints,* Minsk: Vedas, 1997. (in Russian)