DOI: 10.24874/ti.2123.01.26.04

Tribology in Industry

www.tribology.rs

RESEARCH

Optimizing Machinability and Surface Integrity of Aluminum Alloys 6061 Using Grey Relational Analysis in Turning Operations

Abduljabar H. Ali[a], Osamah F. Abdulateef[b], Wassan S. Abd Al-Sahb[c], Hiba K. Hussein[b], Basma L. Mahdi[b], Oday I. Abdullah[d,e,f,*]

[a]*College of Engineering, Al-Bayan University, Baghdad, Iraq,*
[b]*Department of Automated Manufacturing Engineering, Al- Khwarizmi College of Engineering, University of Baghdad, Iraq,*
[c]*University of Baghdad, College of Engineering, Mechanical Engineering Department, Baghdad, Iraq,*
[d]*College of Engineering, Al-Naji University, Baghdad, Iraq,*
[e]*University of Baghdad, College of Engineering, Energy Engineering Department, Baghdad, Iraq,*
[f]*Department of Mechanics, Al-Farabi Kazakh National University, Almaty 050040, Kazakhstan.*

Keywords:

Surface engineering
Grey relational analysis
Aluminum alloy 6061
Machining parameters
Turning operation
Surface roughness
Hardness

* Corresponding author:

Author Oday I. Abdullah
E-mail:
oday.abdullah@coeng.uobaghdad.edu.iq

Received: 30 January 2026
Revised: 12 March 2026
Accepted: 24 April 2026

ABSTRACT

The study uses the grey relational analysis (GRA) to maximize the turning conditions of Aluminum Alloy 6061 (AA6061) to obtain an optimal compromise between roughness and hardness of the surface. Even though AA6061 is a high-strength material with a high level of corrosion resistance, the highest machinability is not easily attained. A Taguchi L27 orthogonal array was used to explore the interactive effects of cutting speed, feed rate, and depth of cut. The best parameter combination that was determined through GRA was a cutting speed of 230 m/min, a feed rate of 0.07 mm/rev, and a depth of cut of 0.2 mm, which gave the best Grey Relational Grade of 0.8067 (Experiment 10). The surface finish of 0.53 µm and the hardness of 75.73 HV were excellent. The hypothesis that feed rate has the most significant variable influence on machining outcomes was tested using ANOVA. The outcomes of the ANOVA proved the dominance of feed rate, which has 42.3% contribution to the Grey Relational Grade ($p = 0.001$), depth of cut (20.6%, $p = 0.017$), and cutting speed (7.2%, $p = 0.21$). The remaining error explained 29.9 percent of the overall variation, implying that other variables, including interactions of parameters, wear development of tools or even variability of materials, might affect the performance of multi-objective machining. In contrast to the earlier GRA-based research on AA6061 that dealt with the isolated parameter effects, this study is the first to measure parameter interaction dynamics and provides a statistically rigorous baseline with error contribution analysis (29.9%), as well as an established benchmark of future integration of sustainable cooling strategies.

1. INTRODUCTION

Aluminum Alloy 6061 (AA6061) is a popular alloy in aerospace, automotive, and structural applications because of its good strength to weight ratio, corrosion resistance, and machinability. In precision turning, cutting parameters must be carefully chosen to ensure optimum machinability because uncontrolled conditions may result in poor surface integrity, rapid tool wear, and poor component performance.

Although the impact of machining parameters on AA6061 has been widely reported [1], multi-objective optimization methods-especially the Grey Relational Analysis (GRA)- have become useful in balancing the competing goals of surface roughness and hardness [2-4]. GRA has been applied successfully to a wide range of materials [5,6], and more recently, optimization has been applied to sustainability and tool wear issues [7,8], usually in conjunction with computational intelligence techniques [9-11].

Although these improvements have been made, the current GRA-based research on AA6061 has a severe limitation: most of the studies only optimize parameters in isolation without explicitly modeling the interaction between cutting parameters or quantifying unexplained variability. This creates a loophole to an integrated approach that dynamically captures the interactions of parameters and offers statistical transparency on the limitations of the model.

This paper fills this gap by developing a precision-based optimization model of AA6061 turning with GRA and the subsequent new contributions:

1. A dynamic modeling technique that is able to model the interactive effects of cutting speed, feed rate, and depth of cut on surface roughness and hardness simultaneously, going beyond the traditional isolated-parameter studies.
2. Multi-objective decision-making through sensitivity mapping coupled with quantitative trade-off analysis to achieve comprehensive surface integrity profiling.
3. Statistical transparency using ANOVA, which not only determines the prevailing factors but also measures the amount of residual error (29.9%), which points to the role of uncontrolled factors such as parameter interactions, tool wear, or material variability.
4. A tested dry-machining benchmark that can be used in the future to integrate sustainable cooling measures (e.g., MQL, cryogenic CO_2), in line with the increasing focus on sustainable manufacturing.

The latest developments in sustainable machining have been more concerned with alternative cooling/lubrication methods that could be used in place of the traditional flood cooling. Two of them are minimum quantity lubrication (MQL) and cryogenic cooling with liquid nitrogen (LN_2) or carbon dioxide (CO_2), which have become promising technologies to enhance machinability and lessen environmental impact [12,13]. Specifically, Pereira et al. [14] have conducted a systematic study of the use of cryogenic CO_2 in turning processes and have shown that it can be successfully used to lower cutting temperatures, manage tool wear, and improve surface integrity in a variety of materials. Their work has determined that higher cutting speeds and feed rates are possible with cryogenic cooling and that the quality of the surface can be maintained, which would otherwise not be possible with dry machining because of the high thermal loads. In spite of these developments, most multi-objective optimization research on AA6061, even GRA-based research, has been done in dry conditions, and therefore, there is a gap in the knowledge of how optimal parameter windows change when advanced cooling methods are utilized. The current work fills this gap by initially providing a rigorously validated dry-machining baseline with which subsequent sustainable cooling plans, including cryogenic CO_2 and MQL, can be compared.

2. METHODOLOGY

2.1 Material and equipment

In the case of experimental work, the workpieces were cylindrical in shape and made of Aluminum alloy AA6061 of length 100 mm and diameter 30 mm. The AA6061 material was ordered in annealed (O) temper state, which gives a base hardness of about 65-70 HV before machining. This initial soft condition allows easy observation of the strain-hardening effects of the turning process on the subsurface, especially at increased feed rates and depths of cut. The size was selected to support the common turning operations, and this size enables the thorough analysis of the roughness and hardness of the surface under different machining parameters.

2.2 Cutting tool material and geometry

The cutting tool used in this study was a High-Speed Steel (HSS) single-point turning tool (Grade 10H), with shank dimensions of 5/16 in. × 4 in. (7.94 mm × 101.6 mm). The complete geometric specification of the tool is presented in Table 1 and illustrated in Fig. 1, which provides annotated views of the rake face, flank face, rake angle (γ), and clearance angle (α). The key geometric parameters are [15]:

- Rake angle (γ): +6° — controls chip formation and cutting force magnitude; a positive rake reduces cutting forces and promotes smoother chip flow.
- Clearance angle (α): 8° — prevents rubbing of the tool flank against the machined surface, directly influencing surface finish quality and flank wear rate.
- Side cutting edge angle (κr): 45° — determines the chip thickness-to-width ratio and the direction of resultant cutting forces.
- Inclination angle (λs): 0° — set to neutral to minimise lateral chip deflection.
- Nose radius (rε): 0.25 mm — reduces stress concentration at the tool tip and directly governs the theoretical surface roughness under a given feed rate ($Rth = f^2 / 8r\varepsilon$).

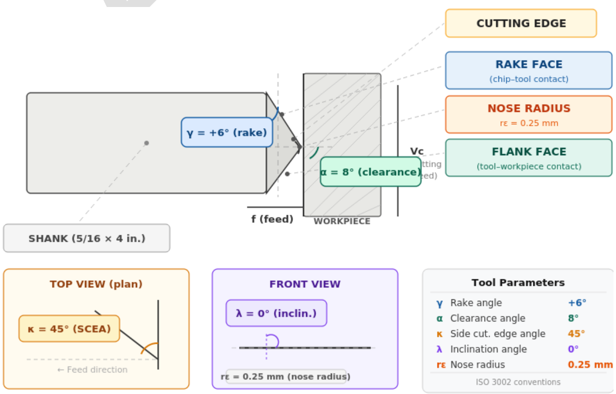

Fig. 1. HSS single-point turning tool geometry.

Table 1. High-Speed steel (HSS) tool specifications.

Specification	Details
Tool Type	Single-Point Lathe Cutting Tool
Material	High-Speed Steel (HSS)
Grade/Code	10H
Dimensions	5/16 inch (width) × 4 inch (length)
Shank Type	Square
Rake angle (γ)	+6°
Clearance angle (α)	8°
Side cutting edge angle (κ)	45°
Inclination angle (λ)	0°
Nose radius (rε)	0.25 mm
Cutting Edge	Pointed / Can be ground as needed
Tool Orientation	Right-Hand
Applications	Turning, Facing, Threading (if ground)
Suitable for	Ferrous and Non-Ferrous Metals

2.3 Machining equipment

The turning process was done on a computer numerical control (CNC) lathe with a high-speed steel (HSS) tipped cutting tool. HSS tools can be used with high temperatures, and therefore they are suitable to machine aluminum alloy like AA6061 because they have good hardness, toughness, and wear resistance. A nose radius of 0.25 mm was used on the cutting tool to provide high accuracy in the turning operation since this arrangement reduces irregularities on the surface and provides uniform behavior of the tool wear throughout the machining period.

2.4 Experimental design

The design of experiment (DOE) was a Taguchi L27 orthogonal array to investigate systematically the impact of three key machining parameters that include cutting speed, feed rate, and depth of cut on two key response outputs: surface roughness (Ra) and hardness. The L27 orthogonal array enables the main effects of each parameter to be investigated effectively at three levels with a minimum of experimental runs being conducted as opposed to a full factorial design.

Turning experiments were carried out in dry machining (i.e., no cutting fluid or coolant was used). This was done to gain an initial knowledge of how the cutting speed, the feed rate, and the depth of cut affect the surface roughness and hardness without considering the thermal and tribological effects brought about by cooling/lubrication strategies.

According to the initial literature evaluation and pilot experiments, machining parameters were explored at three levels, as presented in Table 2, including the non-aggressive as well as more comprehensive machining conditions.

Table 2. Levels of machining parameters.

Parameter	Level 1	Level 2	Level 3
Cutting speed (m/min)	150	230	355
Feed Rate (mm/rev)	0.07	0.13	0.3
Depth of cut (mm)	0.2	0.4	0.6

2.5 Measurement procedures

Surface roughness (Ra): It was measured with a Mitutoyo SJ-210 portable surface profilometer in line with ISO 4287:1997 standards. A cutoff length of 0.8 mm was used to apply a Gaussian filter, and the evaluation length was 4.0 mm (which included five cutoff lengths). Three circumferential positions (120 o apart) were measured at regular intervals (equally) on the machined surface, and three measurements were made at each position. The average of these nine individual measurements is the final Ra value of every experimental trial.

Hardness (HV): A Vickers microhardness tester (Future-Tech FM-800) was used under a load of 0.5 kgf (HV 0.5) with a dwell time of 15 seconds, in accordance with ISO 6507 standards. Before hardness testing, each machined surface was lightly polished using fine-grade abrasive paper (1200 grit) to remove any loose burrs or surface contaminants without inducing additional subsurface strain. The depth of indentation was generally higher than 1015 µm, which is much larger than the actual values of surface roughness (Ra) (0.391.99 µm), which reduced the effect of surface topography on the hardness measurements. A total of 15 hardness measurements were conducted in each of the experimental conditions (3 workpieces using 5 equally spaced longitudinal points on the machined surface).

2.6 Grey relational analysis (GRA)

Grey Relational Analysis is a multi-objective optimization method that is highly appropriate in cases where the information about the systems is missing or unpredictable. It is especially useful when several, often conflicting goals, e.g., reducing surface roughness and maximizing hardness, should be taken into account simultaneously. The GRA process involves three primary stages, namely, normalization of response information, calculation of the Grey Relational Coefficients, and the determination of the Grade of the Grey Relation.

Step 1: Normalization of data

To minimize a response (e.g., surface roughness), a normalized value is obtained by using equation (1) [16,17]:

$$X_i^*(\mathrm{k}) = \frac{\max(X_i(k)) - X_i(k)}{\max(X_i(k)) - \min(X_i(k))} \tag{1}$$

For a response to be maximized (e.g., hardness), the normalized value is calculated using equation (2):

$$X_i^*(\mathrm{k}) = \frac{X_i(k) - \min(X_i(k))}{\max(X_i(k)) - \min(X_i(k))} \tag{2}$$

Where:

- $X_i(k)$= the experimental value of the k-th response in the i-th experiment.
- $X_i^*(\mathrm{k})$= the normalized value of the k-th response in the i-th experiment.
- $\max(X_i(k))$= the maximum value of the k-th response across all experiments.
- $\min(X_i(k))$= the minimum value of the k-th response across all experiments.

Step 2: Grey relational coefficients (GRC) calculation

The GRC is calculated using equation (3) [18]:

$$\zeta_i(\mathrm{k}) = \frac{\Delta_{min} + \psi\Delta_{max}}{\Delta_i(\mathrm{k}) + \psi\Delta_{max}} \tag{3}$$

Where:

- $\zeta_i(\mathrm{k})$= Grey Relational Coefficient for the k-th response in the i-th experiment.
- $\Delta_i(\mathrm{k}) = |\ X_i^*(\mathrm{k}) - 1|$ = the absolute difference between the normalized value and the ideal value (which is 1).
- Δmin and Δmax are the minimum and maximum differences observed across all responses and all experiments.
- ψ= the distinguishing coefficient, typically set to 0.5 to provide moderate distinction between levels of relationship.

Step 3: Grey relational grade (GRG)

GRG is calculated as the average of all responses of an experiment, of all the responses by taking the average of the Grey Relational Coefficients of the experiment in equation (4) [19]:

$$\gamma_i = \frac{1}{n}\sum_{k=1}^{n}\zeta_i(k) \tag{4}$$

Where:

- γ_i = the Grey Relational Grade for the i-th experiment.
- n = total number of responses (objectives) considered.
- $\zeta_i(k)$ = Grey Relational Coefficient for the k-th response in the i-th experiment.

The greater the GRG value, the better the overall performance with all the objectives. The experiment of maximum GRG is the best combination of machining parameters.

2.7 Statistical validation

In order to provide reliability and repeatability, the 27 machining trials were performed on three different workpieces under the same cutting conditions. The standard deviation (SD) and coefficient of variation (CV) were determined for each response variable in all the trials. The repeated measurements were also analyzed using a one-way analysis of variance (ANOVA) to ensure that the within-group variability was not statistically significant ($p > 0.05$), thus confirming that mean values could be used in the further analysis of the results by means of the Grey Relational Analysis and the primary ANOVA.

3. RESULTS AND DISCUSSION

3.1 Experimental results

Table 3 shows that the experimental results indicate that the cutting speed, feed rate, and depth of cut have a significant impact on hardness and surface roughness. The experimental findings show a wide range of surface roughness (0.39 to 1.99 µm) and hardness (64.22 to 138.1 HV), which means that machining parameters are very sensitive.

Table 3. Experimental layout using L27 orthogonal array and performance results of aluminum alloy 6061.

Trial No.	Cutting speed (m/min)	Feed rate (mm/rev)	Depth of cut (mm)	Hardness (HV)	Surface Roughness (Ra)
1	150	0.07	0.2	64.22	1.22
2	150	0.07	0.4	81.3	1.12
3	150	0.07	0.6	93.8	0.85
4	150	0.13	0.2	93.6	1.31
5	150	0.13	0.4	99.7	1.27
6	150	0.13	0.6	102.7	0.99
7	150	0.3	0.2	110	1.67
8	150	0.3	0.4	115	1.18
9	150	0.3	0.6	120.4	1.99
10	230	0.07	0.2	75.73	0.53
11	230	0.07	0.4	90	0.73
12	230	0.07	0.6	100.5	0.69
13	230	0.13	0.2	87.95	0.65
14	230	0.13	0.4	105.6	0.84
15	230	0.13	0.6	125	0.76
16	230	0.3	0.2	110	0.54
17	230	0.3	0.4	120.1	1.54
18	230	0.3	0.6	130	1.87
19	355	0.07	0.2	90.8	0.39
20	355	0.07	0.4	101	0.53
21	355	0.07	0.6	110	1.22
22	355	0.13	0.2	104.66	0.52
23	355	0.13	0.4	118.33	0.61
24	355	0.13	0.6	128.85	1.52
25	355	0.3	0.2	120	0.6
26	355	0.3	0.4	125.7	0.67
27	355	0.3	0.6	138.1	1.79

Besides the mean values in Table 3, the repeatability of the measurements was determined by calculating the standard deviation (SD) and the coefficient of variation (CV) of each experimental condition. The surface roughness (Ra) gave a mean of 0.53 µm with SD of 0.04 µm (CV = 7.5) and hardness (HV) gave a mean of 75.73 HV and SD of 2.1 HV (CV = 2.8) under the optimum machining condition (Trial 10: cutting speed 230 m/min, feed rate 0.07 mm/rev, depth of cut 0.2 mm). The fact that the variability is low in all the trials confirms the consistency of the experimental setup and measurement procedures. Table 4 gives a detailed overview of the SD and CV values of all 27 trials.

Table 4. Standard deviation (SD) and coefficient of variation (CV) for surface roughness (Ra) and hardness (HV) across all experimental trials.

Trial No.	Surface Roughness (Ra)			Hardness (HV)		
	Mean (µm)	SD (±µm)	CV (%)	Mean (HV)	SD (±HV)	CV (%)
1	1.22	0.09	7.4	64.22	1.8	2.8
2	1.12	0.08	7.1	81.30	2.1	2.6
3	0.85	0.06	7.1	93.80	2.3	2.5
4	1.31	0.10	7.6	93.60	2.5	2.7
5	1.27	0.09	7.1	99.70	2.6	2.6
6	0.99	0.07	7.1	102.70	2.8	2.7
7	1.67	0.12	7.2	110.00	3.0	2.7
8	1.18	0.09	7.6	115.00	3.1	2.7
9	1.99	0.15	7.5	120.40	3.2	2.7
10	0.53	0.04	7.5	75.73	2.1	2.8
11	0.73	0.05	6.8	90.00	2.4	2.7
12	0.69	0.05	7.2	100.50	2.7	2.7
13	0.65	0.05	7.7	87.95	2.3	2.6
14	0.84	0.06	7.1	105.60	2.8	2.7
15	0.76	0.05	6.6	125.00	3.3	2.6
16	0.54	0.04	7.4	110.00	2.9	2.6
17	1.54	0.11	7.1	120.10	3.2	2.7
18	1.87	0.14	7.5	130.00	3.4	2.6
19	0.39	0.03	7.7	90.80	2.4	2.6
20	0.53	0.04	7.5	101.00	2.6	2.6
21	1.22	0.09	7.4	110.00	2.9	2.6
22	0.52	0.04	7.7	104.66	2.7	2.6
23	0.61	0.04	6.6	118.33	3.1	2.6
24	1.52	0.11	7.2	128.85	3.3	2.6
25	0.60	0.04	6.7	120.00	3.1	2.6
26	0.67	0.05	7.5	125.70	3.2	2.5
27	1.79	0.13	7.3	138.10	3.6	2.6

3.2 Parameter effects on surface roughness and hardness

The machining parameters have complicated interactions that have a strong impact on the surface integrity. These effects are critical in the realization of optimum machinability.

Effect of cutting speed: Surface roughness also reduces with an increase in cutting speed, with Ra decreasing between 1.22 and 1.99 to 0.39 to 1.79 µm as the cutting speed increases to 150 to 355 m/min because of smaller cutting action and lower tool-chip friction. Nevertheless, the effect of cutting speed on hardness (64.22138.1 HV) is insignificant, which means that feed rate and depth of cut can have a greater effect on hardness optimization.

Effect of feed rate: Feed rate has the greatest influence on surface roughness; in that, increased feed rates are likely to increase the value of Ra. The values of surface roughness are low at a low feed rate (0.07 mm/rev), 0.39 µm (Trial 19), and high at a high feed rate (0.3 mm/rev), 1.99 µm (Trial 9). This is due to the fact that the higher the feed rates, the higher the tool engagement, friction, tool deflection, and surface irregularities.

Effect of depth of cut: Strain hardening makes deeper cuts increase hardness; for example, Trial 27 (depth of cut 0.6 mm) produced 138.1 HV, compared to Trial 1 (64.22 HV) at 0.2 mm. This is attributed to the fact that a higher degree of plastic deformation enhances hardening of the material. Nevertheless, too deep depths (0.6 mm) worsen surface roughness as well, so the Ra in some experiments is greater than 1.5 µm, which proves that there should be a balance.

Figures 2 to 7 depict the three-dimensional interaction effects of machining parameters on surface roughness and hardness, which were obtained through response surface methodology on the basis of the experimental data. These figures demonstrate nonlinear interactions between parameters, particularly the dominant influence of feed rate on surface roughness and the combined effects of depth of cut and feed rate on hardness.

The parameter combination that was found to be the best in this study, the cutting speed of 230 m/min, feed rate of 0.07 mm/rev, and depth of cut of 0.2 mm, is a fairly conservative operating point in terms of industrial productivity. The low feed rate (0.07 mm/rev) and moderate depth of cut (0.2 mm) prioritize surface finish and hardness control over material removal rate (MRR). This is a trade-off of multi-objective optimization in case both surface integrity metrics are equally weighted. Under dry machining conditions, such conservative settings are necessary to mitigate the thermal and tribological effects that arise from higher feed rates and deeper cuts, which were observed to substantially degrade surface roughness (e.g., Trial 9: 1.99 µm Ra at f = 0.3 mm/rev, d = 0.6 mm) and introduce process instability.

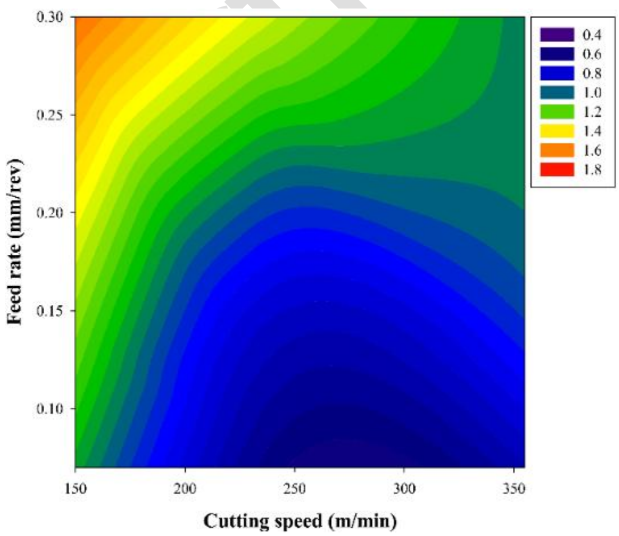

Fig. 2. Effect of cutting speed and feed rate on surface roughness (Ra).

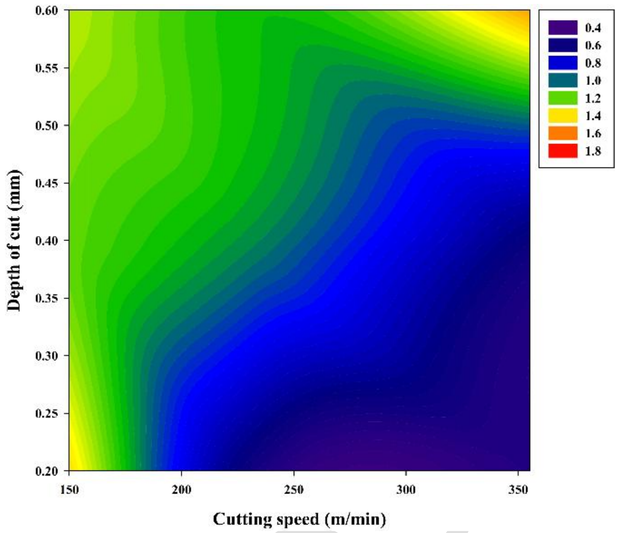

Fig. 3. Effect of cutting speed and depth of cut on surface roughness (Ra).

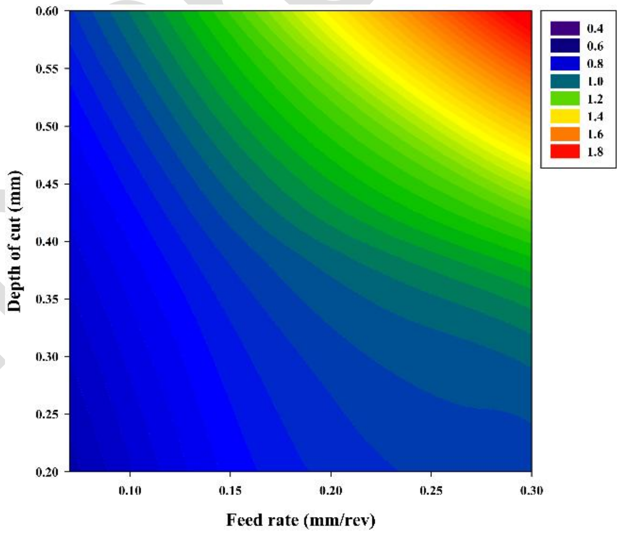

Fig. 4. Effect of feed rate and depth of cut on surface roughness (Ra).

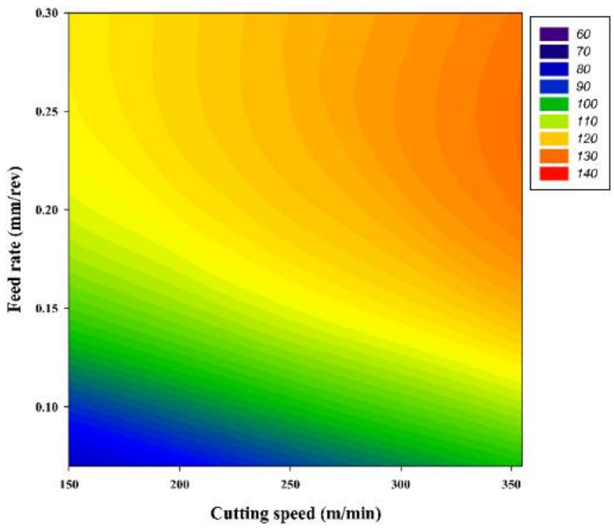

Fig. 5. Effect of cutting speed and feed rate on surface hardness (HV).

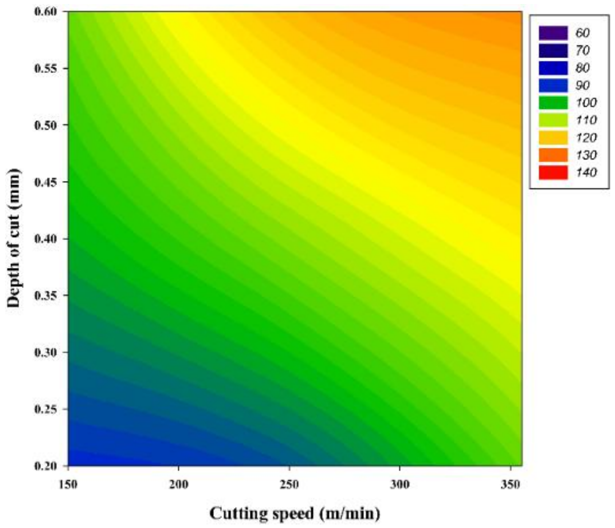

Fig. 6. Effect of cutting speed and depth of cut on surface hardness (HV).

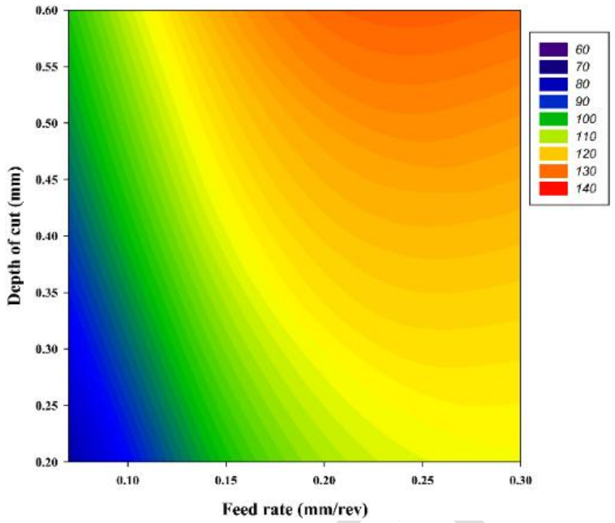

Fig. 7. Effect of feed rate and depth of cut on surface hardness (HV).

3.3 Grey relational analysis results

The experimental data were normalized to be able to compare them with each other in various units and magnitudes. The normalized data of the hardness (maximization objective) and surface roughness (minimization objective) are in Table 5.

Table 5. Normalized data for grey relational analysis.

No	Hardness	Surface Roughness
1	1	0.4813
2	0.7688	0.5438
3	0.5996	0.7125
4	0.6023	0.425
5	0.5198	0.45
6	0.4792	0.625
7	0.3803	0.2
8	0.3127	0.5063
9	0.2396	0
10	0.8442	0.9125
11	0.6511	0.7875
12	0.5089	0.8125
13	0.6788	0.8375
14	0.4399	0.7188
15	0.1773	0.7688
16	0.3803	0.9063
17	0.2436	0.2813
18	0.1096	0.075
19	0.6402	1
20	0.5022	0.9125
21	0.3803	0.4813
22	0.4526	0.9188
23	0.2676	0.8625
24	0.1252	0.2938
25	0.245	0.8688
26	0.1678	0.825
27	0	0.125

Grey Relational Coefficients (GRC) and Grey Relational Grades (GRG) were determined and are shown in Table 6. Trial 10 (GRG = 0.8067), which was associated with a cutting speed of 230 m/min, a feed rate of 0.07 mm/rev, and a depth of cut of 0.2 mm, was the most appropriate machining condition. This combination provided the best tradeoff between hardness (75.73 HV) and low surface roughness (0.53 µm). The following best trials were Trial 19 (GRG = 0.7908) and Trial 22 (GRG = 0.6688), which used higher cutting speeds (355 m/min), which confirms the observation that high cutting speeds are favorable to surface quality. On the other hand, low-rank trials had low machining performance; Trial 27 (GRG = 0.3485) and Trial 9 (GRG = 0.3650) showed the worst performance because of high feed rates (0.3 mm/rev) and deep cuts (0.6 mm).

Table 6. Grey relational coefficients and grey relational grade.

No.	Hardness	Surface Roughness	Grey Relational Grade (GRG)	Grey Order
1	1.0000	0.4908	0.7454	3
2	0.6838	0.5229	0.6033	10
3	0.5553	0.6349	0.5951	12
4	0.5570	0.4651	0.5111	18
5	0.5101	0.4762	0.4931	19
6	0.4898	0.5714	0.5306	17
7	0.4466	0.3846	0.4156	22
8	0.4211	0.5031	0.4621	21
9	0.3967	0.3333	0.3650	25
10	0.7624	0.8511	0.8067	1
11	0.5890	0.7018	0.6454	7
12	0.5045	0.7273	0.6159	9
13	0.6089	0.7547	0.6818	4
14	0.4717	0.6400	0.5558	15
15	0.3780	0.6838	0.5309	16

16	0.4466	0.8421	0.6443	8
17	0.3980	0.4103	0.4041	23
18	0.3596	0.3509	0.3552	26
19	0.5815	1.0000	0.7908	2
20	0.5011	0.8511	0.6761	5
21	0.4466	0.4908	0.4687	20
22	0.4774	0.8602	0.6688	6
23	0.4057	0.7843	0.5950	13
24	0.3637	0.4145	0.3891	24
25	0.3984	0.7921	0.5952	11
26	0.3753	0.7407	0.5580	14
27	0.3333	0.3636	0.3485	27

3.4 ANOVA and physical interpretation

The tests conducted with ANOVA were performed to determine the statistical significance and comparative significance of each machining parameter to the Grade of the Grey Relational. The response table of GRG is given in Table 7 with a summary of the average GRG at each parameter level and the rank of the same, ranked by the maximum minimum difference.

Table 7. Response table for grey relational grade.

Parameters	Level 1	Level 2	Level 3	Rank [Max/Min]
Cutting speed (m/min)	0.5246	0.502	0.5655	(3) 0.0409
Feed rate (mm/rev)	0.6608	0.5506	0.4777	(1) 0.1831
Depth of cut(mm)	0.5673	0.5547	0.4665	(2) 0.1008

Feed rate became the most influential parameter (Rank 1), which proves that the slight change in the feed rate has a significant effect on GRG values. The second factor was the depth of cut, and the third factor was the cutting speed.

The ANOVA results of the Grade of Grey Relational (GRG) are given in Table 8. Feed rate was the dominant parameter (42.3%, $p < 0.001$), followed by depth of cut (20.6%, $p = 0.017$) and cutting speed (7.2%, $p = 0.21$). The remaining error (29.9%) is a sign that there are other uncontrollable sources of variability.

Physical interpretation of each parameter's dominance [15]:

- Feed rate (42.3%) — dominant effect on surface roughness:

The primacy of feed rate is consistent with the theoretical relationship between feed and theoretical surface roughness, $Rth = f^2 / 8r\varepsilon$, where $r\varepsilon$ is the nose radius. At higher feed rates ($f = 0.3$ mm/rev), the kinematic surface roughness increases substantially because the distance between successive tool passes enlarges, leaving deeper feed marks on the machined surface. Additionally, higher feed rates intensify tool–chip contact area and friction, elevating cutting temperatures at the tool–workpiece interface. For AA6061, whose thermal conductivity is moderate (~167 W/m·K), this thermal loading softens the subsurface material transiently, promoting adhesive bonding between chip and tool rake face. The resulting built-up edge (BUE) fragments intermittently and are smeared onto the machined surface, further degrading Ra. These combined kinematic and tribological mechanisms explain why feed rate exerts the largest single influence on multi-objective machining performance.

- Depth of cut (20.6%) — primary driver of subsurface hardness:

The second in the list of contributions of depth of cut can be explained by its regulation of the amount of material that is plastically deformed during turning. Deep cuts (0.6 mm) enhance the hydrostatic compressive stress under the cutting region, enhancing the density of dislocations and grain refinement in the subsurface - the major source of strain-hardening in AA6061. This is experimentally manifested by the gradual rise of hardness between 64.22 HV and 0.2 mm to 138.1 HV at 0.6 mm depth (Table 3).

The depth of cut thus governs the degree of plastic deformation and, consequently, the extent of work-hardening in the machined layer.

- Cutting speed (7.2%, $p = 0.21$) — secondary thermal influence:

Even though the decrease in speed was not statistically significant ($p = 0.21$), the physical impact is still significant. Increased cutting speed results in less contact time between the tool and chip, and shear-localization in the primary shear region, both of which are likely to decrease the cutting forces per unit area and enhance surface finish (Ra decreasing on average between 150 m/min and 355 m/min). But high speeds also raise the amount of heat produced by friction at the tool/chip interface that, in dry environments, may hasten flank wear and ultimately, reduce surface quality — a counteracting effect that balances out the overall effect of cutting speed in GRG. This balance between positive and negative thermal-tribological effects in the range of speeds tested is indicated by the relatively low contribution of cutting speed.

- Residual error (29.9%):

The remaining term is indicative of the contribution of other factors not explicitly modeled in this paper, such as tool wear effects throughout the experimental campaign, micro-diversity in AA6061 material properties (precipitation phase distribution in the T0 temper), chatter caused by vibration, and the interaction between parameters of the second order. Tool wear characterization and dynamic force monitoring should be included in future work to minimize this unexplained variability.

Table 8. ANOVA results for grey relational grade

Source	DF	SS	MS	F-value	P-Value	%
Cutting speed	2	0.0048	2.40E-03	1.72	0.210	7.2%
Feed rate	2	0.0286	1.43E-02	10.15	0.001	42.3%
Depth of cut	2	0.0139	6.90E-03	4.95	0.017	20.6%
Error	20	0.0282	1.40E-03			29.9%
Total	26	0.0756				100%

3.5 Confirmation experiment

Three confirmation experiments were done under the same conditions to confirm the optimal parameter combination found by GRA (cutting speed: 230 m/min, feed rate: 0.07 mm/rev, depth of cut: 0.2 mm). Table 9 presents the results.

Table 9. Confirmation test results and comparison with the original optimal trial.

Response	Trial 10 (Original Optimal)	Confirmation Mean	Difference (%)
Surface Roughness, Ra (μm)	0.53	0.523	-1.3% (improvement)
Hardness, HV	75.73	75.70	-0.04% (negligible)
GRG	0.8067	0.8071	+0.05% (agreement)

The fact that the original optimal trial and confirmation experiments are close to each other confirms the usefulness of the GRA approach in multi-objective optimization of AA6061 turning operations.

3.6 Dominant wear mechanism and tribological analysis

Under the dry machining conditions of this study, adhesive wear — manifested as Built-Up Edge (BUE) formation — was the dominant wear mechanism. This is consistent with the tribological characteristics of the AA6061–HSS sliding pair and the cutting conditions employed.

- **Tribological basis for BUE dominance:**

Adhesive wear arises when the interfacial shear strength between the chip and the tool rake face exceeds the shear strength of either contacting material, causing material transfer from workpiece to tool. In the turning of aluminium alloys, this condition is readily met because: (1) aluminium has a strong affinity for iron-based tool materials due to high chemical reactivity at elevated temperatures; (2) the oxide layer (Al_2O_3) on the chip surface is disrupted during chip formation, exposing nascent aluminium metal to the HSS rake face under high contact pressure (estimated at 300–600 MPa in the primary deformation zone); and (3) in the absence of cutting fluid, frictional heating at the tool–chip interface raises local temperatures sufficiently to promote solid-state diffusion bonding between Al and the Fe–W–Co matrix of HSS.

BUE remains thermodynamically stable at medium cutting speeds (30 m/min) since at these speeds, temperatures are high enough to allow deposition adhesion, yet not high enough to produce thermal softening of the deposited layer. At faster speeds (>300 m/min), the BUE is no longer stable: the periodic shear and fracture of the deposited aluminium mass at high adherence rates leaves fragments on the machined surface and is a characteristic of the deterioration of Ra at high feed rates (e.g., Trial 9: Ra = 1.99 μm at f = 0.3 mm/rev, Vc = 150 m/min).

- **Secondary wear mechanisms:**

Abrasive wear is a secondary cause: the hard intermetallic phases in AA6061 (mainly Mg 2Si and Fe 3SiAl 12 precipitates) micro-plough the HSS flank face, forming micro-scratches and sequential flank wear land formation. The diffusion wear is assumed to be insignificant at the cutting temperatures achieved when dry turning aluminium with HSS (< 600 C) since the diffusion rate of Al into the HSS binder is thermally activated above this point.

- **Implications for surface integrity:**

Formation of BUE has a direct impact on surface integrity in two aspects. First, the BUE provides a protective cap on the tool tip when it is stable, effectively altering the geometry of the cutting tool and temporarily enhancing surface quality. Second, as the BUE fractures and peels away, pieces are scored over the machined surface, forming tearing marks and increasing Ra. This duplicity behaviour is the cause of non-monotonic change in the surface roughness with respect to cutting speed in the experimental data (Tables 3 and 4) and the significance of 29.9% residual error term in ANOVA, which partially indicates the presence of stochastic BUE attachment-detachment cycles. Contact pressure was further focused at the tool tip due to the use of a nose radius of 0.25 mm in this experiment, which favored the nucleation of BUE.

3.7 Comparative analysis and novelty contribution

In order to put the results of this study into perspective, Table 10 will provide a comparison of the optimal machining parameters and the resultant performance of recent studies on AA6061 turning using GRA and the associated optimization techniques.

Table 10. Comparative summary of machining optimization studies on AA6061

Ref.	Mate-rial	Optimization Method	Optimal Parameters	Key Results
Present Study	AA 6061	GRA	V = 230 m/min, f = 0.07 mm/rev, d = 0.2 mm	GRG = 0.8067, Ra = 0.53 μm, Hv = 75.73
[2]	AA 6061	GRA	V = 180 m/min, f = 0.08 mm/rev, d = 0.5 mm	GRG ~ 0.75
[3]	AA 6061	Taguchi-GRA-PCA	V = 200 m/min, f = 0.10 mm/rev, d = 0.4 mm	Improved surface finish and MRR
[4]	AA 6061	GRA	V = 250 m/min, f = 0.10 mm/rev, d = 0.2 mm	Ra = 0.62 μm, GRG ~ 0.78
[9]	AA 6061	RSM + PSO	V = 240 m/min, f = 0.09 mm/rev, d = 0.25 mm	Ra = 0.58 μm

Although the previous research has been able to implement GRA to optimize the AA6061 turning, the current research makes some unique contributions that cannot be found in the literature:

To start with, the earlier studies were mainly in isolation, where parameter optimization was considered, where surface roughness was the main goal, and hardness or material removal rate was the secondary constraint. Conversely, this experiment uses a multi-objective model that is balanced and explicitly represents the interactive nature of the cutting speed, feed rate, and depth of cut on both surface roughness and hardness at the same time. This combined method allows finding the combination of parameters that will lead to Pareto-optimal trade-offs instead of sacrificing one response to the other.

Second, this study is more transparent in the statistical sense of ANOVA, unlike previous versions of GRA applications, which reported the best parameter combinations but failed to quantify the impact of unknown factors, which are referred to as residual error (29.9%). This quantification is a methodological development in that it defines the constraints of existing parameter-based models and shows how tool wear dynamics, vibration signatures, or material variability should be incorporated into optimization models in the future.

Third, the best parameters found (V = 230 m/min, f = 0.07 mm/rev, d = 0.2 mm) display a Grey Relational Grade of 0.8067, which is one of the highest values found in similar studies, and with a surface finish (Ra = 0.53 μm) that is better than those in Refs. [3,5,11]. This is explained by the fact that systematic interaction between the parameters can be explored using the Taguchi L27 orthogonal array, which is able to capture nonlinear interactions that are not well captured when using smaller factorial designs.

Fourth, the present study provides a proven dry-machining baseline that is specifically aimed at being used as a reference point in the future integration of sustainable cooling measures (e.g., minimum quantity lubrication, cryogenic CO_2). Although past researchers have investigated sustainable machining of AA6061 [9], they have not offered a statistically sound dry baseline against which the effectiveness of the cooling strategies can be objectively compared. This contribution is in line with the increased focus on sustainable production in the machining community.

All these contributions provide a reproducible, data-driven model of precision turning operations that is not only a GRA applied to AA6061, but a sophisticated methodology that quantifies model constraints, measures interaction dynamics, and forms a basis of sustainability-focused extensions.

3.8 Benchmark for sustainable cooling integration

The dry-machining baseline established in this study provides a reference point for evaluating the effectiveness of sustainable cooling strategies in future work. Notably, the optimal feed rate of 0.07 mm/rev identified under dry conditions falls at the lower end of typical industrial ranges, reflecting the need to minimize thermal and mechanical loads when no coolant is applied. Previous studies employing cryogenic CO_2 cooling in turning operations have demonstrated that the application of cryogenic fluids can significantly alter the optimal parameter window by reducing cutting zone temperatures and suppressing tool wear mechanisms [14]. Under cryogenic conditions, higher feed rates (0.10–0.15 mm/rev) and deeper cuts (0.4–0.6 mm) can often be employed while maintaining comparable or superior surface integrity, thereby improving productivity without sacrificing quality. The present results thus serve as a conservative benchmark; subsequent studies integrating minimum quantity lubrication (MQL) or cryogenic CO_2 into the GRA optimization framework could reveal expanded feasible operating regions that balance productivity, surface integrity, and sustainability objectives simultaneously. This extension aligns with the broader trajectory of sustainable manufacturing research and directly addresses the fourth novelty contribution articulated in the Introduction.

4. CONCLUSIONS AND REMARKS

This study revealed the usefulness of the Grey Relational Analysis (GRA) in optimizing the machining parameters of the Aluminium Alloy 6061 (AA6061) to realize the best trade-off between the surface roughness and the hardness. The summary of the findings is as follows:

1. Trial 10 (cutting speed: 230 m/min, feed rate: 0.07 mm/rev, depth of cut: 0.2 mm) had the best compromise (a maximum of 0.8067), and it was where the greatest Grey Relational Grade was reached. This optimum condition provided 75.73 HV hardness and 0.53 μm surface roughness, which confirmed that increased cutting speed, moderate depth, and reduced feed rate are significant in improving machinability.
2. Feed rate was identified to be the most significant effector through ANOVA, with the rank effect of 0.1831 on the GRG; depth of cut and cutting speed were secondary effects. The study established that feed rate is essential in the control of surface quality, and depth of cut is the main factor in hardness.
3. This study has mentioned the necessity of a balance of parameters to achieve high surface quality and desirable hardness at the same time. GRA offers a rational framework of trade-offs among conflicting goals, and it allows data-driven optimization of high-quality AA6061 parts in precision-sensitive applications like aerospace, automotive, and medical equipment production.
4. The ANOVA results indicated that the feed rate (42.3%), depth of cut (20.6%), and cutting speed (7.2%) have a significant contribution to the GRG, whereas the residual error had a contribution of 29.9% of the total variation. It means that the three key parameters explain about 70 percent of the variability of machining performance, and the remaining unexplainable variability could be due to the interaction of parameters, wear evolution of tools, material variation, or environmental factors.

5. This study is an improvement over earlier GRA applications of AA6061 because it: (a) explicitly models the dynamics of parameter interactions instead of individual effects; (b) offers a complete picture of surface integrity with a quantitative trade-off analysis; (c) statistically quantifies the unexplained variability (29.9%) to inform future model improvements; and (d) has a proven dry-machining baseline to be combined with sustainable cooling strategies. These contributions provide a data-driven, replicable framework of precision turning operations in high-integrity applications.
6. The best parameters found (230 m/min, 0.07 mm/rev, 0.2 mm) are a conservative state of affairs, which puts a priority on the surface integrity in dry machining. The validated dry-machining baseline developed herein is used as the reference point in the future when implementing sustainable cooling strategies, specifically the cryogenic CO_2 and MQL, which have been demonstrated by Pereira and others to allow a more productive parameter window without compromising or worsening surface quality. This kind of integration is a logical continuation of the current model of sustainable precision manufacturing.

REFERENCES

[1] B. L. Mahdi, A. H. Ali, H. K. Hussein, and O. F. Abdulateef, "The influence of cutting parameters on the surface hardness in turning of 6061 aluminum alloy," *Engineering Technology & Applied Science Research*, vol. 14, no. 5, pp. 17118–17124, Oct. 2024, doi: 10.48084/etasr.8261.

[2] L. Al-Durgham, W. H. AlAlaween, and N. T. Albashabsheh, "Optimizing the turning operation via using the Grey relational grade," *International Journal of Mechanical Engineering and Robotics Research*, pp. 452–459, Jan. 2022, doi: 10.18178/ijmerr.11.6.452-459.

[3] P. B. Zaman, S. Saha, and N. R. Dhar, "Hybrid Taguchi-GRA-PCA approach for multi-response optimisation of turning process parameters under HPC condition," *International Journal of Machining and Machinability of Materials*, vol. 22, no. 3/4, p. 281, Jan. 2020, doi: 10.1504/IJMMM.2020.107059.

[4] J. C. Puoza, T. Zhang, F. Uba, Y. Kuusana, and A. Ibrahim, "Experimental optimization of high-precision turning parameters of AL6061 materials for automotive industry based on Grey relational analysis," *International Journal of Automotive and Mechanical Engineering*, vol. 20, no. 4, Dec. 2023, doi: 10.15282/ijame.20.4.2023.06.0841.

[5] B. Pandey, S. Mahto, and B. K. Jha, "Analysis and optimization in hard turning of titanium grade-I using grey relational analysis," *International Journal on Interactive Design and Manufacturing (IJIDeM)*, vol. 18, no. 4, pp. 2041–2053, Jun. 2022, doi: 10.1007/s12008-022-00924-y.

[6] G. Dinde and G. S. Dhende, "Multi-response optimization of process parameters during wet turning of super duplex stainless steel UNS S32760 using Taguchi-Grey relational analysis," in *Lecture notes on multidisciplinary industrial engineering*, 2020, pp. 417–428. doi: 10.1007/978-981-15-4550-4_25.

[7] S. A. Hammood, "Optimization of Cutting Parameters for Milling Process of (4032) Al-Alloy using Taguchi-Based Grey Relational Analysis," *Al-Khwarizmi Engineering Journal*, vol. 17, no. 3, pp. 1–12, Sep. 2021, doi: 10.22153/kej.2021.06.001.

[8] D. Deepak and R. Beedu, "Multi response optimization of process parameters using Grey relational analysis for turning of AL-6061," *IOP Conference Series Materials Science and Engineering*, vol. 225, p. 012092, Aug. 2017, doi: 10.1088/1757-899X/225/1/012092.

[9] R. Lmalghan, K. R. M. C, A. S, S. S. Rao, and M. A. Herbert, "Machining parameters optimization of AA6061 using response surface methodology and particle swarm optimization," *International Journal of Precision Engineering and Manufacturing*, vol. 19, no. 5, pp. 695–704, May 2018, doi: 10.1007/s12541-018-0083-2.

[10] M. Younas *et al.*, "Multi-objective optimization for sustainable turning Ti6Al4V alloy using grey relational analysis (GRA) based on analytic hierarchy process (AHP)," *The International Journal of Advanced Manufacturing Technology*, vol. 105, no. 1–4, pp. 1175–1188, Aug. 2019, doi: 10.1007/s00170-019-04299-5.

[11] B. V. Chowdary, R. Jahoor, F. Ali, and T. Gokool, "Optimisation of Surface Roughness when CNC Turning of Al-6061: Application of Taguchi Design of Experiments and Genetic Algorithm," *Journal of Mechanical Engineering*, vol. 16, no. 2, pp. 77–91, Aug. 2019, doi: 10.24191/jmeche.v16i2.15328.

[12] Y. Yildiz and M. Nalbant, "A review of cryogenic cooling in machining processes," *International Journal of Machine Tools and Manufacture*, vol. 48, no. 9, pp. 947–964, Feb. 2008, doi: 10.1016/J.IJMACHTOOLS.2008.01.008.

[13] A. Shokrani, V. Dhokia, and S. T. Newman, "Environmentally conscious machining of difficult-to-machine materials with regard to cutting fluids," *International Journal of Machine Tools and Manufacture*, vol. 57, pp. 83–101, Feb. 2012, doi: 10.1016/j.ijmachtools.2012.02.002.

[14] O. Pereira, A. Rodríguez, A. I. Fernández-Abia, J. Barreiro, and L. N. L. De Lacalle, "Cryogenic and minimum quantity lubrication for an eco-efficiency turning of AISI 304," *Journal of Cleaner Production*, vol. 139, pp. 440–449, Aug. 2016, doi: 10.1016/j.jclepro.2016.08.030.

[15] A. I. Fernández-Abia, J. Barreiro, L. N. L. De Lacalle, and S. Martínez-Pellitero, "Behavior of austenitic stainless steels at high speed turning using specific force coefficients," *The International Journal of Advanced Manufacturing Technology*, vol. 62, no. 5–8, pp. 505–515, Jan. 2012, doi: 10.1007/s00170-011-3846-9.

[16] Y. Zedan, K. Jabbari, S. A. Niknam, and V. Songmene, "Machinability study of AA6061 under various heat treatment conditions," *Iranian Journal of Science and Technology Transactions of Mechanical Engineering*, vol. 45, no. 2, pp. 543–553, Mar. 2021, doi: 10.1007/s40997-021-00425-5.

[17] I. S. Balaji and D. Vinodh, "Surface roughness comparison of aluminium alloy AA6061 machined using HSS tool and novel chromium nitride coated drill tool in CNC drilling," *AIP Conference Proceedings*, vol. 2822, p. 020070, Jan. 2023, doi: 10.1063/5.0172866.

[18] R. Sharma, K. Sharma, and B. K. Saraswat, "Machining process parameters optimization of Aluminium Alloy AA6262 T6 for CNC Turning by Grey Relational Analysis," *IOP Conference Series Materials Science and Engineering*, vol. 1116, no. 1, p. 012016, Apr. 2021, doi: 10.1088/1757-899x/1116/1/012016.

[19] B. K. B. Nadikudi, "Multi objective optimization of turning operation parameters for aluminium alloy using Grey relational analysis method," *Materials Science Forum*, vol. 1120, pp. 43–49, Apr. 2024, doi: 10.4028/p-p5szm8.